PEER PBEE Formulation

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Outline

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2. Hazard Analysis
3. Structural Analysis
4. Damage Analysis
5. Loss Analysis
6. Combination of Analyses
Introduction

- **Traditional earthquake design (TED) philosophy:**
  - Prevent damage in low-intensity EQ
  - Limit damage to repairable levels in medium-intensity EQ
  - Prevent collapse in high-intensity EQ

- **TED is necessary but not sufficient as evidenced by:**
  - 1994 Northridge and 1995 Kobe earthquakes (**initial realizations**)
    - Unacceptably high amount of damage, economic loss due to downtime, and repair cost of structures
  - 2009 L’Aquila and 2010 Chile earthquakes (**recent evidences**)
    - A traditionally designed hospital building evacuated immediately after L’Aquila EQ, while ambulances were arriving with injured people
    - Some hospitals evacuated due to non-structural damage and damage to infill walls after Chile EQ
    - Some of the residents rejects to live in their homes anymore despite satisfactory performance according to the available codes
• **First generation PBEE methods:**

Improvement to Traditional Earthquake Design by introducing “Performance Objectives”: Achieve a desired “System Performance” at a given “Seismic Hazard”

<table>
<thead>
<tr>
<th>Hazard Levels (Return Period)</th>
<th>System Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequent (43 years)</td>
<td><img src="image" alt="Table" /></td>
</tr>
<tr>
<td>Occasional (72 years)</td>
<td><img src="image" alt="Table" /></td>
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<tr>
<td>Rare (475 years)</td>
<td><img src="image" alt="Table" /></td>
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<tr>
<td>Very rare (949 years)</td>
<td><img src="image" alt="Table" /></td>
</tr>
</tbody>
</table>

O: unacceptable performance
●: basic safety objective
Δ: essential hazardous objective
♦: safety critical objective

O: unaccept, performance
;
basic safety objective

e: essent, hazardous objective

c: safety critical objective
Introduction

- **First generation PBEE methods - Shortcomings:**

  - Deterministic evaluation of performance: Lack of consideration of uncertainty
  - Evaluation on the element level: Lack of consistency in the determination of the relationships between engineering demands and component performance criteria
  - Evaluation on the element level: Not tied to global system performance
  - Results specific to engineers: Reduced contribution of stakeholders in the decision process
Introduction

- Pacific Earthquake Engineering Research (PEER) Center PBEE:
  - Improvement of first generation PBEE by introducing:
    - Calculation of performance in a rigorous probabilistic manner: Consideration of uncertainty
    - Performance definition with decision variables which reflect the global system performance
    - Performance definition with decision variables in terms of the direct interest of various stakeholders
  - **Shortcoming**: Mostly used by academia with little attention from practicing engineers
Introduction

- **PEER PBEE (Revisited):**
  - Gaining popularity of probabilistic Performance-Based Engineering Design (PBED) methods
  - PBED methods likely to be used for standard design codes in the near future
  - Necessity to find paths for popularization of the method within the practicing structural engineering community
  - **Objective:** Explain PEER PBEE methodology in a simplified manner to reach the broader engineering community
PEER PBEE Formulation

- **Hazard Analysis**: Earthquake hazard during the lifecycle of a building (uncertainty in fault locations, magnitude-recurrence rates, level of attenuation, etc.)

- **Structural Analysis**: Response of the structure to the earthquake hazard (uncertainty in ground motion type, material properties, damping, etc.)

- **Damage Analysis**: Level of damage corresponding to the response of the structure (uncertainty in the damage pattern, history, capacity, etc.)

- **Loss Analysis**: Value of a decision variable (DV, e.g. economic loss) corresponding to damage (uncertainty in damage distribution, variation of components resulting in same damage level, etc.)

**End Product**: Due to the different sources of uncertainty, there is no single deterministic value of DV. Instead, there are multiple values of DV with varying probability.
Hazard Analysis

- First analysis stage in PEER PBEE formulation

- A natural hazard is a threat of a naturally occurring event that will have a negative effect on people or the environment:
  - Earthquakes
  - Volcanoes
  - Hurricanes
  - Landslides
  - Floods or droughts
  - Wildfires

- PEER PBEE considers earthquake hazard (seismic hazard)
Hazard Analysis

- Uncertainty in seismic hazard:
  a. Potential fault locations
  b. Magnitude-recurrence rates
  c. Level of attenuation

- Deterministic Seismic Hazard Analysis (Limited uncertainty consideration: only item “c” above)

- Probabilistic Seismic Hazard Analysis (Complete uncertainty consideration → Preferred method)
Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

1. Determine the potential fault locations

2. Determine the magnitude-recurrence relationships for the faults (rate of each possible magnitude)

3. For all the potential earthquake scenarios (M, R, & L):
   - Using ground motion prediction equations: Calculate the mean and standard deviation ($\mu$ & $\sigma$) of intensity measure (IM) as a function of (M, D)
   - Determine the probability distribution function (PDF) and probability of exceedance (POE) of IM using $\mu$ & $\sigma$
   - Multiply POE with R to determine annual frequency of exceedance (AFE) of IM
Probabilistic Seismic Hazard Analysis (PSHA)

4. Sum AFE from all scenarios to obtain the total annual frequency of exceedance (TAFE) of IM

An easier way of representation of TAFE: Return period of exceedance, $RPE = \frac{1}{TAFE}$
5. From Poisson’s model, calculate POE of IM in $T$ years from TAFE

$$P(IM) = 1 - e^{-\hat{\lambda}(IM)T}$$
6. Calculate probability of IM in T years from POE

\[
p(IM)_{m} = P(IM_{m}) \quad \text{if } m = \# \text{ of IM data points}
\]

\[
p(IM)_{m} = P(IM_{m}) - P(IM_{m+1}) \quad \text{otherwise}
\]
Deterministic Seismic Hazard Analysis (DSHA)

1. and 2. as PSHA
3. For one or only few (generally the most critical) of the potential earthquake scenarios (M, R, & L)
   - Determine the value of intensity measure (IM) as a function of (M, D)
   - Inherent consideration of uncertainty due to the probabilistic nature of ground motion prediction equations
Hazard Analysis

- **Outcome of hazard analysis**: Probability of exceedance (POE) and probability \( p \) of Intensity Measure (IM)

- **Commonly used IMs**:
  - Peak ground acceleration \([\text{PGA}]\)
  - Peak ground velocity \([\text{PGV}]\)
  - Spectral acceleration at fundamental period \([\text{Sa}(T_1)]\)

- **Alternatives for IM** [e.g., Tothong and Cornell (2007)]:
  - Inelastic spectral displacement
  - Inelastic spectral displacement with a higher-mode factor

Reason of common use: Ground motion predictions available
Selecting Ground Motion Time Histories: Compatible with the hazard curve for each intensity level (i.e. each IM value)

- Adequate number of GMs to provide meaningful statistical data in the structural analysis phase
- GMs compatible with the magnitude and distance pair which dominates the hazard
- Use of unscaled GMs whenever possible
- Separation of unscaled ground motions into bins: Performed once and used for consecutive cases
Structural Analysis

- Second analysis stage in PEER PBEE Formulation
- A computational model of the structure:

  Uncertainty in
  - Mass (e.g. variation in live load)
  - Damping (e.g. epistemic uncertainty in damping models)
  - Material characteristics (e.g. strength, ultimate strain)

- Nonlinear time history simulations with ground motions from hazard analysis
Potential variables in analyses:

- Ground motion
- Mass
- Damping ratio
- Damping model
- Strength
- Modulus of elasticity
- Ultimate strain
Determination of **important** variables: Tornado diagram analysis (Lee and Mosalam, 2006)
**Structural Analysis**

- **Determination of important variables**: Tornado diagram analysis (Lee and Mosalam, 2006)

- Determine the variables with negligible effect on the structural response variability and reduce the number of simulations by eliminating unnecessary sources of uncertainties.
Structural Analysis

- Remember Hybrid Simulation (from yesterday’s workshop)
  - In some cases, hybrid simulation can be an alternative to the nonlinear time history simulations
  - For example, elimination of the simulations for the uncertainties in material characteristics

For a specific support structure configuration:
1) Variation in $c$
2) Variation in ground motion

Investigation of the Effect of support structure properties on the seismic response of electrical insulator posts using real-time hybrid simulation (RTHS)
Structural Analysis

- **Structural analysis outcome**: Engineering Demand Parameter (EDP)
- **Local parameters**: e.g. element forces & deformations
- **Global parameters**: e.g. floor acceleration & interstory drift

**Different EDPs for different damageable groups:**
- Axial or shear force in a non-ductile column
- Plastic rotations for ductile flexural behavior
- **Floor acceleration**: non-structural components
- **Interstory drift**: structural & non-structural components

**Peak values** of the above EDPs
Separate treatment of **global collapse** since its probability does not change from a damageable group to the other

**Methods of global collapse determination**

**Method I:** Scaling a set of GMs for each intensity level

**Global collapse:** Unrealistic increase of EDP corresponding to a small increase in IM

**Probability of global collapse for an intensity level:**

\[
p(C|IM) = \# \text{ of GMs leading to collapse} / \# \text{ of GMs}
\]
Separate treatment of **global collapse** since its probability does not change from a damageable component to the other.

**Methods of global collapse determination**

**Method II**: Use of unscaled GMs

**Global collapse from pushover**: Determine EDP at this point = \( EDP_f \)

GMs leading to collapse
Methods of global collapse determination

**Method II**: Use of unscaled GMs

- **a)** \( p(C|IM) = \# \text{ of GMs leading to collapse} / \text{total \# of GMs} \)

- **b)** \( p(C|IM) = \text{shaded area} \)

Lognormal distribution

\[
E(x) = \mu \\
V(x) = \sigma^2
\]

\[
f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}
\]
Progressive Collapse: A realistic representation of collapse in OpenSees

- Start from main code
- Italicized text executed outside of OpenSees
- Check for dangling nodes, floating elements, and element loads and masses
- Identify nodal kinetics at separation
- Track collapsed element motion until impact
- Remove dangling nodes
- Remove floating elements
- Delete element/node loads
- Remove element
- Update nodal masses
- Update structural model, time step, and solution parameters
- Identify location, compute force, duration, and mass redistribution
- Check for dangling nodes, floating elements, and element loads and masses
- Update structural model, time step, and solution parameters
- Identify location, compute force, duration, and mass redistribution
- End back to main code

Talaat & Mosalam (2008)
Progressive Collapse: A realistic representation of collapse in OpenSees

Integration time step $i-1$

Integration time step $i$

IP displacement

OOP displacement

Displacement history

Failure Curve (symmetric about x and y axes)

http://opensees.berkeley.edu/wiki/index.php/Infill_Wall_Model_and_Element_Removal
**Outcome of Structural Analysis:** Probability of each value (index i) of each EDP (index j) for each hazard level (index m): $p(\text{EDP}_j^i | \text{IM}_m)$
Outcome of Structural Analysis: Probability of each value (index i) of each EDP (index j) for each hazard level (index m): \( p(EDP_j^i | IM_m) \)
**Damage Analysis**

- **PEER PBEE objective**: Performance definition in terms of the direct interest of not only engineers, but also various stakeholders
- **Damage analysis**: Third analysis stage to achieve this objective
- **Damage analysis objective**: Estimate physical damage (i.e. Damage Measure, DM) at the component or system levels as functions of the structural response
- **DMs**: Typically defined in terms of damage levels corresponding to repair measures needed to restore components of a facility to the original conditions (other definitions are possible)
- **DM definition example**: Repair with epoxy injections (**light**); Repair with jacketing (**moderate**); Element replacement (**severe or collapse**)
Damage Analysis

- Differences in path of achieving the same EDP & uncertainty in capacity: A specific value of EDP corresponds to various DMs with different probabilities

Uncertainty in damage analysis

FEMA-356

- If PR<0.01 → DM = IO
- If 0.01<PR<0.02 → DM = LS
- If 0.02<PR<0.025 → DM = CP

Examples:

- PR = 0.005 → DM = IO with p=100%
- PR = 0.015 → DM = LS with p=100%
- PR = 0.022 → DM = CP with p=100%
- PR = 0.030 → DM = Collapse with p=100%
Damage Analysis

**FEMA-356**
- $PR = 0.005 \rightarrow DM = IO$ with $p=100\%$
- $PR = 0.015 \rightarrow DM = LS$ with $p=100\%$
- $PR = 0.022 \rightarrow DM = CP$ with $p=100\%$
- $PR = 0.030 \rightarrow DM = Collapse$ with $p=100\%$

**PEER-PBEE**
- $PR = 0.005 \rightarrow DM = IO$ with $p=70\%$, $DM = LS$ with $p=20\%$, $DM = CP$ with $p=18\%$, $DM = Collapse$ with $p=2\%$
- $PR = 0.015 \rightarrow DM = IO$ with $p=15\%$, $DM = LS$ with $p=60\%$, $DM = CP$ with $p=20\%$, $DM = Collapse$ with $p=5\%$
- $PR = 0.022 \rightarrow DM = IO$ with $p=5\%$, $DM = LS$ with $p=15\%$, $DM = CP$ with $p=60\%$, $DM = Collapse$ with $p=20\%$
- $PR = 0.030 \rightarrow DM = IO$ with $p=2\%$, $DM = LS$ with $p=12\%$, $DM = CP$ with $p=21\%$, $DM = Collapse$ with $p=65\%$

**Note:** Probability values are chosen arbitrarily for PEER-PBEE
Tool used in damage analysis:

**Fragility function**: POE of a DM for different values of an EDP

![Diagram showing fragility curves for different levels of damage]

- **DM1** (e.g. Light)
- **DM2** (e.g. Moderate)
- **DM3** (e.g. Severe)
Fragility function determination:
- Analytical simulations
- Experimental simulations (Hybrid simulation or shake table tests)
- Generic functions based on expert opinion (not preferred)

Damageable parts of a structure are divided into damageable groups:
- Each damageable group consists of components that are affected by the same EDP in a similar way
- The components in a group have the same fragility functions
- **Example:** Bohl (2009) used 16 different groups for a steel moment frame building including: (1) the structural system, (2) the exterior enclosure, (3) drift-sensitive and (4) acceleration-sensitive non-structural elements, and (5) office content for each floor
Outcome of Damage Analysis: Probability of each DM value (index k) for each value (index i) of each EDP (index j): \( p(DM_k | EDP_i^j) \)

for \( k = 1 : \# \text{ of DM levels} \)

\[
p(DM_k | EDP_i^j) = P(DM_k | EDP_i^j) \quad \text{if } k = \# \text{ of DM levels}
\]

\[
p(DM_k | EDP_i^j) = P(DM_k | EDP_i^j) - P(DM_{k+1} | EDP_i^j) \quad \text{otherwise}
\]
Loss Analysis

- Last (Fourth) analysis stage in PEER PBEE Formulation
- **Damage information obtained from damage analysis**: Converted to the final decision variables (DVs)
- **Commonly utilized DVs**:
  - Fatalities
  - Economic loss
  - Repair duration
  - Injuries
- **Distribution of damage within the damageable group**: A specific value of DM corresponds to various DVs with different probabilities
- **Economic loss or repair cost as DV**: Uncertainty originating from the economical values, e.g. fluctuation in the market prices, is included
Tool used in loss analysis: Loss function: POE of a DV for different damageable groups and DMs

\[ P(DV \mid DM) \]

\[ \gamma \times \lambda: \text{loss functions} \]
\[ \gamma: \text{# of DM levels} \]
\[ \lambda: \text{# of damageable groups} \]
**Loss Analysis**

**Loss function for collapse:**
- Krawinkler (2005) assumed a lognormal distribution for $P(DV|C)$
- The expected value can be assumed as the total cost of the structural and nonstructural components of the facility
- **Following factors can be considered as sources of variance:**
  - Lack of information about all the present structural and non-structural components
  - Lack of monetary value information about the components
  - Fluctuation in market prices
Total probability theorem:

Given $n$ mutually exclusive events* $A_1, \ldots, A_n$ whose probabilities sum to 1.0, then the probability of an arbitrary event $B$:

$$p(B) = p(B|A_1)p(A_1) + p(B|A_2)p(A_2) + \ldots + p(B|A_n)p(A_n)$$

*Occurrence of any one of them automatically implies the non-occurrence of the remaining $n-1$ events
**Combination of Analyses**

**PEER PBEE combination of analyses:** based on total probability theorem

**End product:**

- **POE of the n\textsuperscript{th} value of the DV of the facility**

\[
P(DV^n) = \sum_m P(DV^n|IM_m) p(IM_m)
\]

**Hazard Analysis**

- Probability of no-collapse & of collapse

\[
P(DV^n|IM_m) = P(DV^n|NC, IM_m) p(NC|IM_m) + P(DV^n|C) p(C|IM_m)
\]

**Structural Analysis:**

- **Probability of no-collapse & of collapse**

\[
P(DV^n|NC, IM_m) = \sum_j P(DV^n_j|NC, IM_m)
\]

**Loss Analysis:**

- Loss function for collapse

\[
P(DV^n|NC, IM_m) = \sum_i P(DV^n_j|EDP^i_j) p(EDP^i_j|IM_m)
\]

**Damage Analysis**

\[
P(DV^n_j|EDP^i_j) = \sum_k P(DV^n_j|DM^i_k) p(DM^i_k|EDP^i_j)
\]

**m:** index for IM

**j:** index for damageable groups (DG)

**i:** index for EDP

**k:** index for DM

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Probabilistic Performance-based Earthquake Engineering, University of Minho, Guimarães, Portugal, October 3-4, 2012
Combination of Analyses

Facility Definition: Location and Design

Hazard Analysis

Outcome:

Loss curve: POE of different values of DV

Decision about Design and Location
**Combination of Analyses**

**Remark:** *Loss, damage, & structural* analyses results are summed in a straightforward manner. However, integration of the *hazard* analysis into the formulation does not take place in such a way because of the presence of damageable groups and collapse and non-collapse cases.

**Straightforward equation in case of a single DG and no collapse:**

\[
P(DV^n) = \sum_m \sum_i \sum_k P(DV^n|DM_k) \cdot p(DM_k|EDP^i) \cdot p(EDP^i|IM_m) \cdot p(IM_m)
\]

**Direct resemblance to the PEER PBEE framework equation:**

\[
\lambda(DV) = \int \int \int G(DV|DM) \cdot dG(DM|EDP) \cdot dG(EDP|IM) \cdot d\lambda(IM)
\]

\(\lambda\): Mean Annual Frequency (MAF), \(G\): Conditional probability
Remark: POE of the DV in case of collapse, \( P(DV|C) \), is not conditioned on the IM, whereas the POE of the DV in case of no collapse, \( P(DV|NC,IM_m) \), is conditioned on the IM because:

- **No collapse** case consists of different damage states and the contribution of each of these damage states to this case changes for different IMs. This is not the situation for **collapse** case.

- For example, loss function for slight damage has the highest contribution for a small value of IM, whereas the loss function for severe damage has the highest contribution for a large value of IM.
Combination of Analyses

Variation in the formulation: Replace POE (\(P\)) with expected value (\(E\))

End product: \(E(DV^n) = \sum_m E(DV^n|IM_m) p(IM_m)\)

\(E(DV^n|IM_m) = E(DV^n|NC, IM_m) p(NC|IM_m) + E(DV^n|C) p(C|IM_m)\)

\(E(DV^n|NC, IM_m) = \sum_j E(DV^n_j|NC, IM_m)\)

\(E(DV^n_j|NC, IM_m) = \sum_i E(DV^n_j|EDP^i_j) p(EDP^i_j|IM_m)\)

\(E(DV^n_j|EDP^i_j) = \sum_k E(DV^n_j|DM_k) p(DM_k|EDP^i_j)\)

\(m: index\ for\ IM\)
\(j: index\ for\ damageable\ groups\ (DG)\)
\(i: index\ for\ EDP\)
\(k: index\ for\ DM\)
Combination of Analyses

**Variation in the formulation:** Replace POE (P) with expected value (E)

**Outcome:** Expected value of the decision variable

**Instead of**
**Loss curve:**

\[ P(DV) \]

**Decision variable (DV)**
Combination of Analyses

Variation in the formulation: Consider a single IM value, $IM_1$

End product: POE of the $n^{th}$ value of the DV of the facility

$$P(DV^n) = P(DV^n|IM_1)\left[p(IM_1) = 1.0\right]$$

Hazard Analysis

$$P(DV^n|IM_1) = P(DV^n|NC, IM_1)p(NC|IM_1) + P(DV^n|C)p(C|IM_1)$$

Structural Analysis: Probability of no-collapse & of collapse

$$P(DV^n|NC, IM_1) = \sum_j P(DV^n_j|NC, IM_1)$$

$$P(DV^n|NC, IM_1) = \sum_j P(DV^n_j|NC, IM_1)$$

Loss Analysis: Loss function for collapse

$$P(DV^n|NC, IM_1) = \sum_j P(DV^n_j|NC, IM_1)$$

$$P(DV^n|NC, IM_1) = \sum_j P(DV^n_j|NC, IM_1)$$

$$P(DV^n_j|NC, IM_1) = \sum_i P(DV^n_j|EDP^i_j)p(EDP^i_j|IM_1)$$

Loss Analysis

$$P(DV^n_j|EDP^i_j) = \sum_k P(DV^n_j|DM^k)p(DM^k|EDP^i_j)$$

Damage Analysis

$m$: index for IM = 1

$j$: index for damageable groups (DG)

$i$: index for EDP

$k$: index for DM
How can an engineer use PEER PBEE method?

1. Evaluation of a traditional code-based design in a performance-based probabilistic approach. This application is appropriate in the current state of traditional code-based design if the engineer wants to introduce performance-based enhancements to the mandatory code-based design.

2. Evaluation of the performance of an existing structure or the outcome of different retrofit interventions.

3. Use of the methodology directly as a design tool, e.g. for decision-making amongst different design alternatives. This type of application is expected to gain widespread use when the probabilistic PBED methods start to be employed as a standard design method.
Thank you