Computer Modelling of the Spatial Variability of Material Parameters Within a Historical Masonry Arch Railway Bridge
DECLARATION

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Year: 2012

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ABSTRACT

The analysis of historic structures is often hindered by inherent uncertainties within the materials and composition of the structures. This is especially true in the case of 19th century Central European masonry arch bridges, as many are still in use today, some 150-200 years later. The presented work exhibits a case study on one such bridge in Rohrbach bei Mattersburg in Burgenland, Austria. Due to limited construction documents and available experimental tests, little was known about the structure. The present work sought to develop a set of analyses using 2D finite element analysis models utilizing the ATENA 2D and SARA Studio reliability software to model the uncertainty and heterogeneity within the various materials of these structures.

Building upon the current state of the art regarding masonry and infill material variability, material parameters and variations were assigned to each material within the bridge models. The models were first analysed with homogeneous material parameters in a deterministic analysis of the structural behaviour. A series of different models were developed to observe the behaviour of the structure under different assumptions and varying levels of stiffness influence from the spandrel walls. These models were validated through comparison with 3D models, developed in a concurrent study, and experimental deflection data. The analyses were limited to 2D by current software, as the final step was a stochastic analysis implementing random fields to generate material parameter distributions according to the obtained variation parameters.

The results provided further insight into the increased vulnerability of structures to concentrated inhomogeneities and weak areas within the structure. The stochastic models generally exhibited failure mechanisms at lower loading levels than the deterministic model, and the failure mechanisms were traced to weak areas within corresponding material parameter distributions throughout the structure. The conservative deterministic analysis provided a structural safety margin 25 times higher than current service loads, while the stochastic analysis lowered that safety margin to around 20 times higher than current service loads. The results showed that the Rohrbach bei Mattersburg bridge was very conservative in design and is still very strong. Nevertheless, insight was obtained into the structural failure mechanisms of the bridge. The spatial variability of the material parameters was seen to influence the development of different failure mechanisms dependent upon the location of weaknesses within the structural elements.

Keywords: historical structures, masonry arches, railway bridges, structural analysis, nonlinear finite element analysis, spatial variability of material parameters, random fields, stochastic simulation of structures.
ABSTRAKT

Nedomyslitelné nejistoty v materiálových vlastnostech a vnitřní struktuře historických staveb jsou často jedním z faktorů, které způsobují numerickou analýzu těchto konstrukcí. Toto tvrzeň se naplňuje zejména v případě středoevropských zděných klenbových mostů z 19. století, z nichž mnohé jsou dodnes, po 100 až 150 letech, stále používány. Obsahem předkládané práce je případová studie na jednom z takovýchto mostů, který se nachází v obci Rohrbach bei Mattersburg v oblasti Burgenland v Rakousku. Vzhledem k tomu, že se dochovala pouze malá část stavební dokumentace a měli jsme k dispozici výsledky jen několika experimentálních zkoušek, naše informace o konstrukci byly velmi omezené. Cílem práce bylo provést sérii 2D konečněprvkových výpočtů s využitím softwarového balíku ATENA 2D a SARA Studio, které by zohledňovaly nejistoty a heterogenity v jednotlivých materiálech zkoumané konstrukce.

Parametry a jejich variabilita byly definovány pro jednotlivé materiály v modelu konstrukce (zdivo, násyp) na základě provedené studie současného stavu poznaní o prostorové variabilitě vlastností zdiva a násypových materiálů. Chování konstrukce bylo nejprve analyzovalo deterministicky za předpokladu homogenního prostorového rozdělení parametrů jednotlivých materiálů. Za tímto účelem byla vytvořena série modelů, které umožňovaly sledovat chování konstrukce při různých předpokladech modelování a různých úroveň ztužení v důsledku spolupůsobení poprsních zdí. Tyto modely byly validovány porovnáním s výsledky 3D modelů, které byly analyzovány v rámci souběžně probíhající čině studie, s experimentálně zjištěnými průhyby. Analýzy byly provedeny ve 2D s ohledem na to, že v posledním kroku byla prostorová proměnnost materiálových vlastností zohledněna pomocí náhodných polí a používaný software umožňuje vytvářet tato pole pouze v rovině.


Klíčová slova: historické konstrukce, zděné klenby, železniční mosty, analýza konstrukcí, nelineární analýza metodou konečných prvků, prostorová variabilita materiálových parametrů, náhodná pole, stochastická analýza konstrukcí.

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1. **INTRODUCTION**

Accurate modelling of historical bridges and structures using today’s sophisticated computer modelling softwares can be integral to understanding structural behaviour and in the diagnosis of structural interventions. The task, however, tends to be much more complicated on historical structures than on modern structures. This is due to the highly variable construction methods and materials used, and the relative lack of available information regarding the construction maintenance on the structures throughout their existence. Before the 20\textsuperscript{th} century, there were few if any widespread construction codes regulating procedures and materials. Therefore, many structures varied due to location, available resources, and people working on the structure. For these reasons, it is a difficult task to be able to properly model the variation within the materials and the inner composition of historical structures. In addition, many historical structures are under special protection and have such cultural value that physical testing is either limited or prohibited. Therefore, more advanced methods of numerical and non-destructive analysis must be undertaken.

The first goal of this work was to create a two-dimensional finite element model of a specific 19\textsuperscript{th} century historical masonry arch railway bridge in Southeastern Austria that is currently still in use by the Austrian Railways. Due to very limited information about the structure and a limited amount of documentation and in-situ tests, the material properties and exact geometry were relatively unknown. This study, proposed by and worked in conjunction with software developers and engineers at Cervenka Consulting in Prague, Czech Republic, sought not only to model the structural behaviour and failure of the case study bridge, but to attempt to develop a method to accurately model the inherent inhomogeneities of the historical materials within the bridge.

As is the case with many similar 19\textsuperscript{th} century and earlier masonry arch bridges, the Rohrbach bei Mattersburg bridge has little documentation regarding the actual construction. The process of adding infill atop the substructure of a bridge was often different for every case. The materials used were mainly local gravels and soils, and the distribution of aggregate within the material was not clearly regulated, nor documented. This distribution, however, may play an integral role in the structural behaviour of the bridge, and irregular or imbalanced distributions could lead to weak points within the bridge. The role of stochastics in this case study will be to create an array of different material property inputs using random field distributions throughout the various elements of the structure. The results obtained from several tests conducted with the stochastic inputs would provide a range of both the behaviour of the bridge and expected distribution of inhomogeneities. The behaviour and distribution results would then be compared to the results from the available in-situ tests conducted on the bridge.

The study will follow the scientific method through first interpreting existing experimental test data and conducting a literature review of previous works to understand similar structures. Based on that data, hypotheses will be made to complete the creation of numerical models, and the models will be
compared with existing test results. Then, deterministic failure analysis will be conducted on varying models of the structure to obtain insight into the possible failure mechanisms of the structure given the assumed inputs. After the failure analysis, stochastic analysis will attempt to model the material inhomogeneities and observe their effect on the behaviour of the structure. This analysis would attempt to compensate for the lack of precise material data. The models will be tested under self-weight, service loading, and until failure. The nonlinear behaviour within the randomized models will be analyzed to observe the effects of the heterogeneous materials on the behaviour. The distributions of inhomogeneities within the materials would then also be compared to those discovered with in-situ georadar scans on the structure for validation.

This process will utilize the nonlinear finite element analysis software ATENA 2D and the structural analysis and reliability assessment software SARA, both developed by Cervenka Consulting. The aim is to test and develop a successful analysis regiment to advance the current state of modelling of historic structures for conservation or structural analysis projects where integral data about the structure is limited. The models created in this study, limited to 2D modelling due to software restrictions on the application of random fields, will be developed concurrently with 3D models of the same structure being developed in the works of Milia, 2012.
2. ROHRBACH BEI MATTERSBURG RAILWAY BRIDGE

2.1 Bridge Information

The case study presented in this work is the Rohrbach masonry arch railway bridge that is located within the municipality of Rohrbach bei Mattersburg in the Burgenland region of Southeastern Austria. The bridge services the Mattersburg railway line of the Austrian Federal Railway Authority (OEBB). The masonry arch bridge consists of 5 three-centered arches with a rise of approximately 2.0 m and a span of approximately 6.0 m. The bridge is connected to earthen dams via abutments on either end, as seen below in Figure 2-1, and in Annex A, S03.

![Figure 2-1: Rohrbach bei Mattersburg Bridge Elevation Design Drawing](Rohrbach bei Mattersburg Plan set, 2012)

The location of Rohrbach bei Mattersburg within the scope of the Mattersburg Railway connecting Wiener Neustadt, Austria and Sopron, Hungary can be seen in Figure 2-2(a). The bridge, having appeared to have spanned only over a river at the time of construction as suggested in Figure 2-1, now spans also over two single lane roadways, as seen in Figure 2-2(b). The bridge is located approximately 250 meters Southeast of the Marz-Rohrbach train station platform.

![Figure 2-2: Rohrbach bei Mattersburg Bridge Location](a) Mattersburg Railway (Dengg and Mendlig, 2012) (b) Bridge Location Aerial Image (Google, 2012)
2.2 History

The bridge was constructed between the years 1845 and 1847 AD, on the railway line connecting the cities of Vienna, via Wiener Neustadt, in present-day Austria with Sopron in present-day Hungary. At the time of construction, the area around Rohrbach bei Mattersburg and the larger town of Mattersburg was under the rule of the Kingdom of Hungary. From the time of construction until 1920, at which time the German-speaking lands in Hungary were given back to Austria following the dissemination of the Austro-Hungarian Empire, Rohrbach was situated within the administrative district of the city of Sopron. (Dengg and Mendlig, 2012) Therefore, all historical documents of this bridge were archived in Sopron. History, however, partially neglected the Rohrbach bei Mattersburg bridge because it was overshadowed by the Mattersburg viaduct during the time of their construction. The population and media were mainly concerned with the Mattersburg viaduct due to its much higher rise than the Rohrbach bei Mattersburg bridge, therefore, historical documents concerning the Rohrbach bei Mattersburg bridge were overshadowed by the Mattersburg viaduct.

The bridge was originally designed and constructed to support one set of rails for each direction of railway traffic, resulting in the width of 8.85 meters. (Dengg and Mendlig, 2012) The bridge, with its deep vault, can be seen in Figure 2-3(a). After construction, only one set of rails was laid to handle traffic in both directions. This condition still exists, with only one set of rails being present on the bridge, but the rails are not positioned symmetrically in the center. The rails are eccentrically positioned slightly to the Southwest of the central position, as seen in Figure 2-3(b).

![Elevation View of Bridge](image1)

![Eccentric Rail Position](image2)

Figure 2-3: Bridge Elevation and Rail Position (Dengg & Mendlig, 2012)

The unique features of the structure, including the drain locations and eccentric rail position will become important later during the analyses of the structure and present additional complexities.
2.3 Railway Usage

The Rohrbach bei Mattersburg railway bridge is currently under the authority of the Mattersburg regional railway, as part of the Austrian Federal Railways (OEBB). Current service on this line includes one hourly passenger railcar in each direction: northwest toward Mattersburg and southeast toward Loipersbach-Schattendorf. Two types of railcars, OEBB types 5047 and 5147, are currently servicing this route and can be seen below in Figure 2-4. The two trains are very similar, with 5147 being a two-railcar variant of the 5047.

![Railcars](image)

Figure 2-4: Service Railcars

In order to accurately model the service loading of the bridge using a finite element model, proper geometry and train weights were needed. This information would allow for accurate distributions of the train loads into the bridge structure. The dimensions of the axles from a single railcar (type 5047) can be seen in Figure 2-5.

![Axle Dimensions](image)

Figure 2-5: Railcar Geometry (Dengg and Mendlig, 2012)

There are 8 wheels situated on 4 axles through the length of each railcar. The 4 axles are separated into 2 sets of 2 axles, with the separation between the two sets being 18.6 m on center. Additional information regarding the trains can be seen in Table 2-1.
Table 2-1: Service Train Information (OEBB)

<table>
<thead>
<tr>
<th>Train Type</th>
<th>Length (m)</th>
<th>Railcars (persons)</th>
<th>Capacity (persons)</th>
<th>Weight (tonnes)</th>
<th>Maximum Speed (km/h)</th>
</tr>
</thead>
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<tr>
<td>Type 5047</td>
<td>25</td>
<td>1</td>
<td>68</td>
<td>45</td>
<td>120</td>
</tr>
<tr>
<td>Type 5147</td>
<td>49</td>
<td>2</td>
<td>124</td>
<td>89</td>
<td>120</td>
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</tbody>
</table>

For purposes of later modelling, the weights of the trains 5047 and 5147 will be increased to 50 tonnes and 100 tonnes, respectively. With regards to the 2D modelling, the loads distributed to each axle were considered. For a single railcar, the full train single axle load adopted for use in the models was 0.1226 MN. These values would give a baseline estimate for service loading, which would then be used to compare all other loadings.
3. TESTING

Multiple in-situ non-destructive and destructive tests have already been conducted on the structure. Technicians and engineers from BOKU University in Vienna, the Austrian Central Institute for Meteorology and Geodynamics, and the Austrian Institute of Technology have all conducted experimental tests in cooperation with the Austrian Railway Authority (OEBB). The data obtained from the in-situ tests was used for coordination and validation of the computer models. The available data from the test conducted will be presented in this chapter.

3.1 Ground Penetrating Radar

Ground penetrating radar, or georadar, is a non-destructive testing method allowing for subsurface investigation without the need for excavation or other destructive techniques. This technique allows for the identification of interfaces between different elements or layers of material within the analyzed area. The process essentially transmits radar waves of specified wavelengths, typically between 100 MHz and 1600 MHz, through a material with a transmitting antenna and measures the radar waves reflected back to a receiving antenna, generally attached adjacent to the transmitting antenna. From the amplitude and time-delay of the reflected signals, radargrams are produced. (Jol, 2009) These radargrams record location and time and create a two-dimensional section along the axis travelled by the transmitter/receiver apparatus. Radargrams allow a specialist to identify different materials, and interfaces between materials, due to the patterns visible on the radargram. There is typically a very high information density in these results, and highly trained experts must conduct the interpretations of the data. The results of ground penetrating radar (GPR) tests do not give exact material parameters, rather general trends of material properties and changes within material properties. If actual values are desired, sample cores of materials should be taken. However, this is a destructive test and is not always possible on structures of historical value.

With the known propagation speed of the transmitter and the depth range that is specified for the test, the amplitudes of the reflection signal at various locations beneath the transmitter can be obtained. The amplitude values are a measure of the reflectivity of the surface at a certain depth. High amplitude values would indicate a highly reflective environment, such as from dense solids or structures like walls and stones. Low amplitude values, on the other hand, would indicate a high absorbing environment, such as soils or clay. (Jol, 2009) This amplitude difference would allow for the discovery of the interface between the infill and vault of a masonry bridge.

GPR tests were conducted on the Rohrbach bei Mattersburg bridge by a team of specialists from the Austrian Central Institute for Meteorology and Geodynamics (ZAMG) on 1 February 2012. These GPR tests were conducted as an archaeological survey of the interior composition and structure of the bridge material. The GPR tests were conducted atop the bridge along the length and can be identified as area A in Figure 3-1. Two arches were also analysed with GPR, and are shown as B and D in Figure 3-1. (Jol, 2009)
The results of the GPR tests showed no abnormalities suggesting the presence of voids or washouts within the analysed area. A distribution of the raw data was produced by the specialists at ZAMG for each 0.5 m section of the structure, as shown in Figure 3-2(a). The aforementioned properties were then interpreted from the raw data and compiled to produce the prospective material map as seen in Figure 3-2(b), showing where each material was expected to be at a specified depth.
The combinations of the sections created at different depths were then combined to create a three-dimensional image of the investigated depth range. In Figure 3-3, an additional three-dimensional model was created by the specialists at ZAMG showing an interpretation of the material distribution throughout the bridge.

![Figure 3-3: 3-D Interpretation of Ground Penetrating Radar Results (Totschnig et al., 2012)](image)

The results from the GPR tests were used to confirm the general geometry of the various materials within the structure in the numerical model creation, since not all dimensions were available from the surviving construction documents. During the course of the present study, the three-dimensional fields produced from the raw GPR data will be used to compare the results of the random fields generated through stochastic analysis of the numerical models. This would be useful to help determine if the spatial variability properties obtained are reasonable assumptions compared to experimental distributions and variability.

### 3.2 Laser Vibrometer Tests

Further testing on the bridge was conducted in the form of laser vibrometer tests to determine the deformation of the bridge under service loading. A team from the Austrian Institute of Technology conducted the laser vibrometer tests on 20 March 2012. These tests, conducted during a 4-hour window between 10:00 and 14:00, included the passage of 8 separate trains, which run hourly passages in each direction across the bridge. These tests included 5 passes of the OEBB type 5047 single-car passenger train and 3 passes of the type 5147 double-car passenger train, both types having been described in detail in Section 2.3.

The laser vibrometer test was limited in the fact that it measured only one point: the vertical displacement of the midpoint of the keystone at the intrados of the middle arch. The laser was positioned perpendicular to the point at the top of the arch, as seen in Figure 3-4, and directly beneath the midpoint between the two rails of the railway track. A reflective sheet was placed at the arch to provide the most accurate return signal.
A laser vibrometer records the vibration of a surface through measuring the phase difference between a reference laser beam, and the beam aimed at the test surface. The difference between the two beams, named the Doppler frequency shift \( f_d \), is then found. (Polytec, 2012) This shift is measured as a voltage, which is proportional to the velocity of the surface in the same plane as the beam, as seen below in Equation 1.

\[
f_d = 2 \times \frac{v}{\lambda} \quad \text{[Hz]}
\]

The relationship between the Doppler frequency shift with velocity is therefore seen, where \( v \) is the surface’s velocity and \( \lambda \) is the wavelength of wave emitted by the vibrometer. Displacement can also be measured with the vibrometer through the relationship between the frequency, velocity and displacement seen below in Equation 2.

\[
v = 2\pi \times f \times s \quad \text{[m/s]}
\]

Since the velocity, \( v \), is a function of the frequency, \( f \), and displacement, \( s \), the movement of the arch surface could be tracked and various attributes of its motion could be measured. (Polytec, 2012) The vibrometer evaluated the maximum velocity and maximum deflection caused at this point for every pass of a train. The speed of the train was then calculated from the time shift of the maximum displacements and the known distances between railcar axles. The results from the 8 passing train tests can be seen in Table 3-1.
Table 3-1: Laser Vibrometer Test Data (Lechner, 2012)

<table>
<thead>
<tr>
<th>Train</th>
<th>Train Number (OEBB Route)</th>
<th>Time of Passage</th>
<th>Direction</th>
<th>Number of Railcars</th>
<th>Train Velocity</th>
<th>Max. Velocity</th>
<th>Max. Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7730</td>
<td>10:26</td>
<td>Mr</td>
<td>1</td>
<td>83</td>
<td>11.9</td>
<td>0.334</td>
</tr>
<tr>
<td>2</td>
<td>7719</td>
<td>10:34</td>
<td>Ls</td>
<td>1</td>
<td>55</td>
<td>17.6</td>
<td>0.341</td>
</tr>
<tr>
<td>3</td>
<td>7734</td>
<td>11:26</td>
<td>Mr</td>
<td>1</td>
<td>81</td>
<td>9.1</td>
<td>0.332</td>
</tr>
<tr>
<td>4</td>
<td>7723</td>
<td>11:34</td>
<td>Ls</td>
<td>2</td>
<td>50</td>
<td>4.4</td>
<td>0.305</td>
</tr>
<tr>
<td>5</td>
<td>7738</td>
<td>12:25</td>
<td>Mr</td>
<td>2</td>
<td>82</td>
<td>9</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>7727</td>
<td>12:33</td>
<td>Ls</td>
<td>1</td>
<td>61</td>
<td>14</td>
<td>0.347</td>
</tr>
<tr>
<td>7</td>
<td>7729</td>
<td>13:02</td>
<td>Ls</td>
<td>2</td>
<td>74</td>
<td>4.7</td>
<td>0.352</td>
</tr>
<tr>
<td>8</td>
<td>7742</td>
<td>13:27</td>
<td>Mr</td>
<td>1</td>
<td>88</td>
<td>17.7</td>
<td>0.349</td>
</tr>
<tr>
<td>9</td>
<td>7731</td>
<td>13:35</td>
<td>Ls</td>
<td>1</td>
<td>58</td>
<td>6</td>
<td>0.347</td>
</tr>
</tbody>
</table>

The information in Table 3-1 shows data from trains passing in both directions, the first being toward Marz-Rohrbach (Mr) and Mattersburg, and the other toward Loipersbach-Schattendorf (LS) and Sopron. The train velocities were calculated from the laser vibrometer displacement and velocity data. The displacement data found, showing an average vertical displacement of -0.34 mm at the keystone, will be used for the validation of the finite element models to ensure that they are able to recreate actual displacements found in this in-situ investigation.

### 3.3 Arch Masonry Core Sampling

Upon request of the Austrian Federal Railways, an investigation into the compressive strength of the arch masonry of the Rohrbach bridge was conducted. This investigation, conducted on 06 October 2010 by specialists from the company Bautechnische Pruef- und Versuchsanstalt GmbH, utilized three drilled core samples to identify the average compressive strength of the arch masonry. The locations of the three drilled cores can be seen in Figure 3-5.

![Figure 3-5: Core Sample Locations](Eisenhut, 2010)
Once the samples were obtained, the brick and mortar materials were separated and the individual
compressive strengths were measured through testing with an ATH 50 compression testing machine.
Three brick samples from each drilled core were cut into cylinders with a diameter of 50 mm and
height-to-diameter ratio of 1. Seven mortar samples from each drilled core were cut into cubes of side
length 20 mm, and the strength results were adjusted to compensate for the small size of the samples.
The results of the compressive strength tests are shown in Table 3-2.

Table 3-2: Core Sample Compression Tests

<table>
<thead>
<tr>
<th></th>
<th>Bulk Density</th>
<th>Compressive Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MN/m3</td>
<td>N/mm2 (MPa)</td>
</tr>
<tr>
<td><strong>Brick Compression Test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Brick Strength</td>
<td>0.0157</td>
<td>30.60</td>
</tr>
<tr>
<td>Average Standard Deviation</td>
<td>0.0002</td>
<td>15.06</td>
</tr>
<tr>
<td>Average Coefficient of Deviation</td>
<td>0.0124</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Mortar Compression Test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Mortar Strength</td>
<td>--</td>
<td>2.48</td>
</tr>
<tr>
<td>Average Standard Deviation</td>
<td>--</td>
<td>2.22</td>
</tr>
<tr>
<td>Average Coefficient of Variation</td>
<td>--</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The results from these core sample compression tests provided valuable information that will be used
later in the study to create an accurate experimental mean value for material properties within the
computer models. The strength of the two materials within the masonry would be combined to
produce single representative parameter values to allow for the modelling of the heterogeneous
material as a homogeneous continuum for the deterministic analysis.
4. CONSTRUCTION METHODS

When modelling any historical structure, it is integral to have a full understanding of the design and construction processes leading to the realization of the final state of the structure. It is important to understand the technology available at the time of construction to be able to create model geometry in an appropriate fashion. The construction process can also have a significant effect on the final structure, due to settlements and self-weight issues. For example, if a large masonry structure was built over a period of several years, the mortar in the first elements of the structure was able to reach a higher strength or full strength before being loaded with the entire self-weight of the structure. This process gives the structure more strength, as the elements are given time to readjust to their position and any additional settlement takes place gradually as more weight is added. If the entire self-weight of the structure were to be added at one single time, the stresses within the structure would be extremely high and cracking would most likely occur quickly. This action may cause nonlinear behaviour in models of larger structures, and if all of the elements are added at one time the structure could crack. Therefore, analysing the behaviour of models under self-weight must be conducted carefully.

Due to the relatively limited amount of information present about the Rohrbach bridge at the time of model construction, a literature review was conducted to better understand the construction methods and technologies used in mid-19th century Europe. These methods were compared to the known information of the Rohrbach bridge, and conclusions were made to the probable construction methods used. The typical method to construct masonry arch railway bridges throughout Europe at the time was to first construct the piers, then set up a scaffolding, usually from timber, to lay out the shape of the arch vaults and support them during the curing of the mortar. (Auroville Earth Institute, 2012) The scaffolding, or centering as it is called, was typically left in place after the vaults were finished, through the addition of the infill. Among various bridges with various spans, centering was removed either immediately after completion, 15-20 days, or 1-2 months afterward. (Brencich & Morbiducci, 2007) This length of time affects the final position of the bridge, as mentioned earlier, and the curing of the mortar takes at least 28 days before it has any substantial strength. Premature removal of the centering support could change the structural behaviour of the entire bridge and could result in elastic deformations due to compaction of voids in the joints or stress redistributions. Typical setups for centering supports can be seen in Figure 4-1.
The importance of construction methods within numerical analysis could rise due to elastic behaviour resulting from the self-weight of the structure. In the case that the self-weight causes cracking or other elastic behaviour within a numerical model, steps could be taken to apply the self-weight in an element-by-element manner so as to allow for settlement and mimic the construction methods, thus removing the unrealistically premature elastic behaviour.

The presence of haunching was also discovered during excavations and tests of several 19th century bridges throughout Germany and Italy. Haunching refers to additional arch masonry material that is added to connect the vaults of the bridge and lessen the angle of the structural arch so as to give the bridge more strength. This material was often added on site during the construction depending on the contractor. Due to the typical lack of available documents on these structures, haunching is often not visible or known until the bridge is deconstructed. The works of Brencich & Morbiducci, 2007, and Huerta, 2006, show the added benefit attributed to haunching on the vault strength and the ability for the vault to be constructed thinner. Haunching was also found to be present in a majority of the bridges examined, so analyses of the Rohrbach bridge will also be conducted assuming that haunching is present. The dimensions were obtained through typical measurements found in Austrian bridges in the works of Brencich & Morbiducci, 2007, and from information present on the available construction documents. A comparison of a selected bridge with a haunching layout and the Rohrbach bei Mattersburg bridge is shown in Figure 4-2.
As seen in the diagram in Figure 4-2(a), the haunching is located just beneath the drain (circled) of the infill. The drain located on the construction documents in Figure 4-2(b) appeared to be in line with the typical geometry mentioned in the historic manuals for the Austrian bridges mentioned in Brencich & Morbiducci, 2007, so haunching for the bridge was assumed to be situated as such. The considerations observed from this investigation were used in the later formulation of the numerical model geometries.
5. **DETERMINISTIC ANALYSIS**

The first goal of this work with the case study of the Rohrbach bei Mattersburg bridge was to develop, validate, and analyse a two-dimensional finite element analysis model utilizing the ATENA 2D software. The fundamental basis of the software and the various assumptions made in the various models will be discussed before the resulting models are presented and analysed to observe structural behaviour and failure mechanisms.

5.1 **ATENA 2D Finite Element Software**

The software used to conduct the models in this study was the ATENA 2D nonlinear finite element analysis software developed by Cervenka Consulting in Prague, Czech Republic. ATENA 2D was designed for nonlinear finite element analysis (FEA) of structures, with a more specific design to simulate concrete and reinforced concrete behaviour. Although the program specializes in reinforced concrete analysis, the software is also adept at modelling other types of structures, including masonry and timber structures.

The software’s capabilities for 2D analysis of continuum structures include a graphic user interface that allows for access to the inner ATENA solution core. The continuum states that the software is able to model are: plane stress, plane strain, and rotational symmetry. In addition to the graphic user interface, the user is able to manually edit the finite element input file to allow for advanced control while modelling uncommon or special conditions within the structure. (Cervenka, 2012) This feature, which will be described in more detail during the deterministic analysis, allows for advanced capabilities in the modelling of 3D structures when limited to 2D analysis.

It is also possible to model construction processes through the manual editing of the input file. For the modelling of larger structures, the addition of the entire structural self-weight simultaneously can create unrealistic nonlinear phenomena, as described in Chapter 4. These phenomena, such as cracking or plastic strains, can drastically affect the behaviour of the structure. Therefore, certain elements that are added later in the construction process, such as the vaults and spandrel walls, can be removed from the initial model to allow the self-weight of the supporting elements to settle. The element could then be added to the model later. This process would be most realistic to the loading process experience during construction, as described in Chapter 4.

5.1.1 **Finite Element Solution Methods**

The finite element method is a solution method that allows for the discretization of a certain domain (i.e., a material element) into elements of a finite size, often referred to as a mesh. This numerical method is based on the relation of the degrees of freedom within the discretized elements with displacements, strains, stresses, and external forces through the use of partial differential equations that relate the various parameters. The overall method tracks the behaviour of the domain as a whole through iterative solution methods. For more detailed information on the formulation and theory
behind the finite element analysis, it is advised to consult literature such the ATENA Theory Manual by Cervenka, 2012, and the vast literature available on the topic.

Within the broader scope of the nonlinear analysis, there are multiple numerical solution methods available within FEA able to track the nonlinear behaviour of a material model. The two main numerical solution methods that were used in the present study, Newton-Raphson and arc-length, will be detailed within this section.

5.1.1.1 Newton-Raphson Method

The standard solution procedure for nonlinear problems in finite element analysis is the Newton-Raphson method. This method uses the calculation of interior forces within a body to the external forces by means of a function of the residual forces. These residuals represent the total load after applying a prescribed load increment subtracting the internal forces at the end of the previous load step. (Cervenka, Jendele, & Cervenka, 2012) The stiffness matrix in this method is determined depending on the value of the load level, \( p \), before the current load increment, meaning that the stiffness matrix depends on the deformation. This method is visualized in Figure 5-1.

![Newton Raphson Solution Method](image)

**Figure 5-1: Newton Raphson Solution Method**  (Cervenka, Jendele, & Cervenka, 2012)

The Newton-Raphson method is useful in approximating such structural behaviour as self-weights and additional loadings within the elastic state and also when the behaviour enters a nonlinear, plastic state. (Kabele, 2011) However, this method is not capable of accurately predicting post-peak failure behaviour and should not be used once the structural model begins to exhibit highly nonlinear behaviour and approaches failure. In this case, the arc-length method should be used.
5.1.1.2 Arc-Length Method

The arc-length method is based on a more recent theory that has exhibited increased success in assuring accurate and realistic results comparing to the traditional Newton-Raphson methods. The arc-length method is often used when failure loads are sought, as it is able to properly follow the post-peak, snap-back and snap-through phenomena observed in highly loaded materials. (Cervenka, 2012) The Newton-Raphson method is unable to capture these phenomena, and can result in inaccurate results where the solution diverges from realistic behaviour. While the Newton-Raphson method applies constant loading increments within each analysis step, the arc-length method observes the relationship between loading patterns and displacement through each load step. The method then comes back to fix the load and displacements at the end of each load step. This procedure can be seen in Figure 5-2.

![Figure 5-2: Arc-Length Solution Method (Cervenka, Jendele, & Cervenka, 2012)](image)

This advanced computational ability, and the ability to model post-peak behaviour, made the arc-length method desirable for the latter steps of the FEA computations. The models created in the present study will therefore begin analysis attributing self-weight body forces and low-level train loadings by utilizing the Newton-Raphson method. As the load increases above typical service loadings toward expected failure loadings, however, the arc-length method will be utilized.

5.1.2 Continuum State Definition

With regards to the modelling of three-dimensional structures within the constraints of a two-dimensional continuum, different continuum states were available within ATENA 2D to model the load distributions within the structure. The two continuum states that will be utilized in the models of the
Rohrbach bei Mattersburg bridge will include the plane strain and plane stress states. The governing equations and basic theory of these states is presented below in 5.1.2.1 and 5.1.2.2.

### 5.1.2.1 Plane Strain State

The primary continuum state to be used in the modelling process will be the plane strain state. This state assumes that a body acts as an extruded, thick structure. A body under this continuum state is based on a two-dimensional cross-section, as shown in Figure 5-3(a), which is acted upon only by loads parallel to the plane of the cross-section. The three-dimensional extrusion is then assumed to be consistent throughout, as seen in Figure 5-3(b), and all loads are also uniform throughout the thickness of the extruded body.

The plane strain state is best used to model simple bodies that experience similar loading patterns throughout their depth. This is often used when modelling retaining walls and other structures that extend for a substantial distance along one axis while keeping the same general cross section. (Kabele, 2011) The strain tensor, $\varepsilon$, and stress tensor, $\sigma$, of a body under the plane strain state are seen below in Figure 5-4.
As visualized in the strain tensor matrix, the plane strain state does not account for strains in the x-direction. Due to the assumption that the structure is an extruded cross-section, only strains in the y- and z-directions are to be realized.

5.1.2.2 Plane Stress State

The second continuum state to be utilized in the analysis is that of the plane stress assumption. A plane stress state assumes that the body in question is a planar thin structure that is much larger in two axial directions than the third, such as a wall, as represented in Figure 5-5.

A plane stress state assumes also that all the loads acting on the body occur within the same plane as the thin structure. This assumption reduces the number of variables that need to be considered and allows for simpler calculations to be made. (Kabele, 2011) The stress tensor, \( \mathbf{\sigma} \), then reduces to two variables, as seen in Figure 5-6(a). The strain tensor, \( \mathbf{\varepsilon} \), of a body under the plane stress state is also seen below in Figure 5-6(b).
Due to the complex nature and assumptions made in this study, both the plane stress and plane strain continuum states were modelled within different elements of the bridge structure. This is typically not conducted in 2D models, as it requires special attention that adjacent elements assuming different continuum states were interacting properly, and not transferring stresses in undesired areas. For this reason, multiple monitoring points observing displacements and stresses in corresponding nodes of various elements were used throughout the analyses. This was conducted to ensure that the behaviours of such elements were not inappropriately connected. This procedure allowed for the separation of the roles of the elements in the behaviour of the structure in accordance with the intrinsic assumptions of the models.

5.1.3 Constitutive Models

Two main types of constitutive models were used in the definition of the materials of the model. The first, a combined fracture-plastic model that was developed and implemented into ATENA, was used to model masonry materials. The other, a common Drucker-Prager plasticity model for soils, was used for the bridge infill material. The assumptions and theory behind these constitutive models will be outlined in this section.

5.1.3.1 Fracture-Plastic Constitutive Model

The main constitutive model used within the material elements of the models in this study was the combined fracture-plastic model developed by Cervenka & Papanikolaou, 2008. This model combines constitutive models for both tensile and compressive behaviour, thus being able to predict fracturing and plastic deformation within a body. The fracture model is based on an orthotropic smeared crack model and uses the Rankine tensile failure criterion. The plastic hardening and softening behaviour is modelled using the Menetrey-William plasticity criterion simulating the crushing of concrete. (Cervenka & Papanikolaou, 2008) The model not only allows for both types of failures to occur...
simultaneously, due to recursive substitution allowing both models to progress independently, but it can handle instances where crack closure also occurs. (Cervenka, Jendele, & Cervenka, 2012) This combined fracture-plastic model has been integrated into the ATENA 2D software as various constitutive models.

The specific constitutive model used for the masonry materials in the bridge models in this project was the \textit{CC3DNonLinCementitious2} constitutive model, with the definitions seen in Figure 5-7. This figure describes the behaviour assumed for the combined model with regards to the compressive strength, $f_c$, tensile strength, $f_t$, and compressive elastic modulus, $E_c$, of the material.

At the time of analysis, this was the most advanced version of the fracture-plastic model integrated into the ATENA 2D software. This constitutive model was deemed appropriate due to the relative similar properties of concrete and masonry, in that both materials are very strong in compression and very weak in tension. This model, in particular, assumes an incremental formulation of hardening behaviour within the material before the compressive strength is attained. For more detailed information regarding the theory and specific model, please consult the ATENA Theory Manual. (Cervenka, Jendele, & Cervenka, 2012) The pre-set assumptions within the algorithms for the \textit{CC3DnonLinCementitious2} model originally designed for concrete can be applicable to masonry materials, albeit, under proper conditions when discretion is used. This model will be used as the constitutive model for the finite elements within the regions defined for the masonry materials within the bridge. This includes the pier support masonry, the vault brick masonry, the spandrel wall stone masonry, and the parapet brick masonry.

The intrinsic heterogeneity of masonry, using this method, would not be completely properly modelled under normal analysis. In this instance, material properties accounting for the combination of both brick and mortar would result in a homogeneous continuum. It was understood that this method is not realistic in nature, but is often used in this field as a way to incorporate masonry elements into finite element analysis. The later goal of this study is to apply random fields of material parameters within

![Stress-Strain Relationship](image1.png)  ![Failure Surface](image2.png)

Figure 5-7: Fracture-Plastic Constitutive Model (ATENA 2D, 2012)
the masonry elements in an attempt to accurately model this heterogeneity caused by the combination of bricks and mortar, and observe its effect on the behaviour of the structure.

### 5.1.3.2 Drucker-Prager Plasticity Model

Although the exact material properties within the infill are not explicitly known, it can be deduced from previous investigations and literature, discussed in Chapter 4, that typical infill materials used in the time of construction of the Rohrbach bei Mattersburg bridge behave as soil. Therefore, a typical Drucker-Prager plasticity model will be assigned to the infill material. This model is based on the Drucker-Prager yield criterion of plastic materials with low to zero tensile strength, and is often used for the prediction of soil failure within the field of geotechnical engineering. The yielding function for this surface is described below in Equation 3, where $I_1$ represents the first invariant of the Cauchy stress tensor, and $J_2$ represents the second invariant of the deviatoric stress tensor.

\[
F_{DP}^\sigma (\sigma_{ij}) = \alpha I_1 + \sqrt{(J_2)} - k = 0
\]  

(3)

The yielding function depends on two parameters, $\sigma$ and $k$, which control the shape of the yield surface and whose values are functions of the friction angle, $\phi$, and cohesion, $c$, properties of the material. (Cervenka, Jendele, & Cervenka, 2012) This function and further visualization of the application of the constitutive model can be seen below in Figure 5-8.

![Drucker-Prager Plasticity Constitutive Model](image)

**Figure 5-8: Drucker-Prager Plasticity Constitutive Model (ATENA 2D, 2012)**

This specific constitutive model, within ATENA 2D, is named `CC3DDruckerPragerPlasticity`. The ATENA Theory manual may be consulted for more information regarding this particular model. (Cervenka, Jendele, & Cervenka, 2012) For the sake of brevity, and that this model is quite widely known within structural mechanics, the specifics will not be further discussed in this study.
5.2 Model Definition

5.2.1 Model Creation and Assumptions

Due to the specific and complex nature of the goals of this analysis, certain assumptions were required when creating the model, regarding the behaviour of the structure and interactions between materials. The relative lack of construction plans and documentation available at the time of model creation also created the need to make certain assumptions.

The two-dimensionality of the model was the first assumption. This was mainly due to certain restrictions on the integration of ATENA and the SARA stochastic analysis software that will be discussed later. The implementation of the stochastic fields to model the inhomogeneities within the historical materials was, at the time of analysis, only available for use within ATENA 2D. For this reason the models were limited to two dimensions, and therefore, appropriate assumptions had to be made. Validation of the 2D models was conducted with 3D models being created concurrently with the sister software, ATENA 3D, to model the same structure. The work regarding the three-dimensional models can be seen in the work of Milia, 2012.

Since the models were created in ATENA 2D, the models would return displacements representing average displacements through the transversal axis of the structure, perpendicular to the rail direction (depth of bridge in the models). This was due to the usage of plane strain continuum states in the elements with substantial transversal depth throughout the structure and which absorb the main service loads of the structure. The infill, vault masonry, and pier support masonry were modelled using this plane strain assumption.

The material elements whose transversal depth was not of primary concern to the overall structural behaviour, and whose primary axis lays in the longitudinal direction of the bridge, were then modelled as plane stress elements. The spandrel wall and parapet were modelled as plane stress, due to the fact that the behaviour through the transversal dimension is not integral to the modelling that is being conducted. The modelling did not account for the transversal stress transfer of forces from the infill outward against the spandrel wall. This transversal action had, however, been observed to contribute to certain failure mechanisms in arch bridges in such works as that of Fanning, Boothby, & Roberts, 2001. For the sake of this 2D analysis, however, this phenomenon was not considered.

Additional assumptions made for the models are as follows:

- Masonry vaults are of equal thickness throughout the structure.
- The bridge is symmetric about its centerline, so models created are half-thickness of the bridge to reduce computational complexity.
- Train loading was modelled as static point loads derived from two axles of a 50-ton OEBB 5047 railcar. This corresponded to a 0.1266 MN load per axle.
• Axle loads were modelled as geometrical restraints prevented modelling each wheel.
• Train loading was transmitted to steel rail elements to distribute loading to the infill.

Due to the fact that the main goal of the study concerned the spatial variability in the main elements of the bridge, the role of the rail bed sleepers and gravel was not considered in the load distribution. A steel rail element that was modelled with a representative geometry of two typical rails was used to distribute the point loads from the train through the infill material. Also, in order to verify that there was no shear transfer or interaction between the infill and spandrel wall elements, monitoring points were placed on nodes corresponding to the same location on each element to ensure that the displacements and stresses within the nodes was not equal.

5.2.2 Derivation of Basic Finite Element Model
The geometry of the finite element model was created using information gathered from one existing original construction plan. (Rohrbach Plan set, 2012) Not all dimensions and details required were available within the plan set as the only remaining plan available for analysis appeared to be partially incomplete. An analysis of the high-resolution scan of the document was used in order to derive the additional data. Research into the building methods of similar masonry arch bridges at the time resulted in the creation of the model using the three-centered circle method. (Bencich & Morbiducci, 2007) (Gago, Alfaiate, & Lamas, 2011) The basic geometry adopted for the models can be seen below in Figure 5-9, while the full set of scaled drawings drafted to create the model geometry can be seen in Annex A. Additionally, the drawings within Annex A include section cuts dimensioning the depths of the material elements.

![Figure 5-9: Basic Geometric Layout [mm]](image)

Once the basic geometry was determined and drafted, the models were produced within ATENA 2D using both line and arc creation to model the bridge. Appropriate 2D modelling of a 3D bridge meant that the spandrel wall and infill elements were essentially stacked in front of each other when the
bridge was viewed from the 2D (elevation) model plane. In order to model both the spandrel wall and the infill separately within a single 2D model, the infill model had to be geometrically shifted to another area within the model space. This unusually sophisticated model had to be created with the utmost discretion, as creating two unconnected geometric entities within a single model could present several problems when not properly calibrated. At first glimpse, the infill element within the setup, as seen below in Figure 5-10, appears to be floating in midair above the bridge model without support in the vertical direction. This situation, however, will be dealt with by means of a method outlined in the following Section 5.2.3.

![Figure 5-10: Basic Model Geometry](image)

The boundary conditions, seen below in Figure 5-11(a), were assigned to the elements according to the assumptions made. The bottom interface of the piers, cut at ground level of the original plans, assumes rigid foundations beneath the pier elements. This assumption, made due to the present study's concentration regarding the materials within the above ground structural elements, fixed the base of the piers in both the x- and y-directions. Also, due to the cut of the model only concentrating on the behaviour around the central arch, the edges of the model were fixed in the x-direction. The initial two-axle train loading was applied to the rail element, as seen in Figure 5-11(b), was centered about midspan of the arch.
As described earlier in Section 2.3, the full single axle train load obtained from the train data was equal to 0.1226 MN. It is understood that this train load is theoretically twice that exposed to the half-depth of these models, but this value was used as a conservative baseline, and a later comparison was made to discuss the eccentricity of the train loading. From the train geometry data obtained and the geometry of the 2D model, only two axles of the train would fit on the model at one time. Therefore, two point loads representing two axles of the train were used for the analyses.

Later sections will discuss in further detail the support of the infill material, the attribution of material parameters to the corresponding areas within the model, different variants of the basic geometry, and the train loading calibration to determine the location with the most harmful influence on the structural system.

### 5.2.3 Kinematic Linking of the Infill Element

With regards to the “floating” infill element lacking vertical support discussed in Section 5.2.2, the solution required manually editing the ATENA input file before the first FE analysis step. In editing the input file before each analysis, the two independent geometric bodies could be kinematically connected. This connection was executed using the so-called “master-slave” method within the ATENA 2D program. This method essentially took nodes from different macro-elements corresponding to the same physical node, i.e. overlapping nodes on interfaces, and kinematically linked the two nodes. (ATENA Input File Format, 2012) The procedure was automatically conducted along lines between two adjacent macroelements. To connect the infill to the main model structure, the nodes along the bottom of the infill element had to be kinematically linked to the area along the area of the structure where the infill would lie. This area corresponds to the extrados of the vault and the top of the piers between the vault elements. In order to obtain the FE node numbers along these points...
interfaces, an additional load case was assigned to support the lines along these interfaces, as seen in Figure 5-12.

Figure 5-12: Load Case to Determine Interface Nodes

Since the FE mesh element size and mesh refinement along lines was calibrated in order to ensure that nodes and their distribution would be equal along both interfaces, the nodes could be easily arranged in the proper order and linked. The lines supported in Figure 5-12 show up in the ATENA input file as lists of restrained nodes, as seen in Figure 5-13(a). To link the models via the “master-slave” method, the nodes from one interface were set to act as master nodes to the other interface, thus resulting in both sets behaving in the same manner. The implementation of this connection was set, by default within ATENA for kinematically connecting adjacent macroelements, as a separate connectivity load case, as seen below in Figure 5-13(b).

(a) Support Load Case Definition
(b) Master-Slave Nodal Connectivity

Figure 5-13: ATENA Input File Manipulation Examples
The list of nodes created for the infill-vault interface was added to this pre-existing list before the first step of analysis and the model would then behave as though the two materials are connected. To avoid recursive linking errors, however, nodes in areas such as the middle of the piers had to be treated with more caution. In these areas, nodes from multiple macroelements were already linked, and proper order of the linkage needed to be considered when updating the list to include the nodes from the infill elements.

It must also be noted, that under the conditions set forth by the kinematic linking of the elements as outlined in this section, the connection between the infill and vaulting elements was assumed as a rigid interface. Therefore, the interaction of the friction and slip between the vault masonry and infill soils was not considered. This interaction at the interface, most often modelled under the Mohr-Coulomb failure criterion, was not significant with regards to the goals of the project, being spatial variability of the materials. Therefore, it was decided to model it simply as a rigid connection.

5.3 Material Definition

Perhaps the single most important, and simultaneously most complex, task when modelling historic masonry structures is that of properly defining the material elements. This is due to the variability of past construction methods, the lack of proper construction documentation, and the limitations on testing posed by historical status and/or the current usage of the structures. Therefore, three different methods of investigation were used to derive and justify the material parameters to be input into the models. The first and most accurate method was to derive material parameters from testing results conducted on the actual structure. The second method was to derive the parameters from existing literature, similar numerical models created, and testing results from similar structures. The third, and final method, was to use engineering judgement to define material parameters when the first two methods are not readily available. The following sections will describe each material created for the numerical models, and the origin of the used parameters.

5.3.1 Brick Masonry Vault Material

The clay brick masonry material used in the vaults (arches) of the bridge was the only material with substantial testing data available for the Rohrbach bei Mattersburg bridge. As outlined in Section 3, three main types of tests were conducted on the brick masonry material which allowed for the estimation of the material properties of the masonry as a continuum material. The resulting material properties can be seen below in Table 5-1, along with the integral information regarding the usage of this material in the numerical models created.
Table 5-1: Vault Brick Masonry Material

<table>
<thead>
<tr>
<th>BRICK MASONRY MATERIAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Name</td>
<td>Arch Masonry</td>
</tr>
<tr>
<td>Material Type</td>
<td>CC3DNonLinCementitious2</td>
</tr>
<tr>
<td>Idealisation</td>
<td>Plane Strain</td>
</tr>
</tbody>
</table>

**Basic Properties**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>E</td>
<td>2.08E+03 MPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>ν</td>
<td>0.2</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>f_t</td>
<td>5.24E-01 MPa</td>
</tr>
<tr>
<td>Compressive Strength</td>
<td>f_c</td>
<td>-5.24E+00 MPa</td>
</tr>
</tbody>
</table>

**Tensile Properties**

| Fracture Energy  | G_f   | 2.50E-05 MN/m |

**Miscellaneous Properties**

| Weight Density | ρ     | 1.60E-02 MN/m³ |

**Finite Element Mesh Properties**

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>Quadrilaterals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh Element</td>
<td>CCIsosQuad</td>
</tr>
<tr>
<td>Element Size</td>
<td>0.1524 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>4.2672 m</td>
</tr>
</tbody>
</table>

The brick masonry material was modelled with the combined fracture-plastic model CC3DNonLinCementitious2 as outlined in Section 5.1.3.1, and utilized the plane strain state. The elastic modulus, Poisson’s ratio, and tensile strength of the brick were adopted from the tests performed by Verstrynge et al., 2011 on clay brick masonry of a similar bridge with similar properties as those obtained from the tests on the Rohrbach bei Mattersburg bridge. The compressive strength and weight density were adopted from the results of the mortar and brick compressive tests conducted on the core sample tests by Eisenhut, 2012, and by Schmidt Hammer tests also conducted on the bricks in Brick Compression Strength: Rohrbach, 2012. The fracture energy of the clay brick masonry was obtained from the values presented by Schueremans & Van Gemert, 2005 and Lourenco, 2011.

The finite element mesh properties were defined through the implementation of the basic geometry of the model. The most efficient of the mesh elements in ATENA 2D for use in this model, the quadrilateral element CCIsosQuad, was used with proper element size to obtain the best meshing possible with multiple layers of elements perpendicular to the arc surface. This was conducted so as to best mimic the layout of the brick and would best allow for cracking to occur in a realistic fashion. The thickness of the element was taken to be one half of the total vault depth as derived from the information present during model creation.
5.3.2 Stone Masonry Spandrel Wall Material

The spandrel wall masonry, which was seen in the photo documentation of Figure 2-3(a) to be sandstone masonry with mortar joints, was not previously tested as with the clay brick masonry. Time constraints and the restrictions placed upon the bridge by the OEBB Railway Authority regarding additional tests on the structure prevented further material properties from being obtained through testing. The resulting material properties can be seen below in Table 5-2, along with the integral information regarding the usage of this material in the numerical models created.

Table 5-2: Spandrel Wall Stone Masonry Material

<table>
<thead>
<tr>
<th>SPANDREL WALL MATERIAL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Name</td>
<td>Wall Masonry</td>
<td></td>
</tr>
<tr>
<td>Material Type</td>
<td>CC3DNonLinCementitious2</td>
<td></td>
</tr>
<tr>
<td>Idealisation</td>
<td>Plane Stress</td>
<td></td>
</tr>
</tbody>
</table>

- **Basic Properties**
  - Elastic Modulus: $E = 3.13E+03$ MPa
  - Poisson's ratio: $v = 0.2$
  - Tensile Strength: $f_t = 8.95E-01$ MPa
  - Compressive Strength: $f_c = -8.95E+00$ MPa

- **Tensile Properties**
  - Fracture Energy: $G_f = 2.50E-05$ MN/m

- **Miscellaneous Properties**
  - Weight Density: $\rho = 2.30E-02$ MN/m$^3$

- **Finite Element Mesh Properties**
  - Mesh Type: Triangles
  - Mesh Element: CCIsoQuad
  - Element Size: 0.1524 m
  - Thickness: 1.2192 m

The stone masonry material was modelled with the combined fracture-plastic model CC3DNonLinCementitious2 as outlined in Section 5.1.3.1, but utilized the plane stress state. This was due to the assumption that the transversal stress transfer between the infill and spandrel wall, and the transversal stress distribution within the spandrel wall, were not being considered in the models. The stone masonry appeared to be a sandstone material with lime mortar, so values from literature were obtained from similar structures. The elastic modulus, compression strength and tensile strength of the stone masonry were adopted from ranged established in Ozen, 2006 and Gonzalez et al., 2007. Additionally, Martinez et al., 2003, provided a range of values for the remaining parameters. Values adopted combined the influence of both the stone and lime mortar in the overall behaviour of the masonry.
Due to the geometric restrictions of the spandrel wall elements in the initial models, the FE mesh elements used were triangular CCIsoQuad elements. The sharp angles of the spandrel wall regions resulted in meshing problems with quadrilateral elements; so triangular elements were initially used. The element size was based on the value adopted for the vault brick masonry, so as to allow the two regions to join together well during FE mesh generation. The thickness of the elements was based on the entire thickness of a single spandrel wall observed in the construction documents.

### 5.3.3 Stone Masonry Pier Material

It must be noted that due to relative lack of experimental data, similarities between types of stone masonry, and the time constraints of the project, the pier stone masonry within the structure was also modelled using the material properties adopted for the spandrel wall stone masonry. The parameters assigned to the spandrel wall masonry were reasonable for an array of such stone masonries. Also, since the end goal of the present study was to vary the material properties of the elements and compensate for the uncertainty within the materials, these assumptions were deemed suitable. The stress state and FE mesh parameters were, however, modelled independently to keep in line with the model assumptions, and the adopted parameters can be seen below in Table 5-3.

<table>
<thead>
<tr>
<th>PIER MATERIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Name</td>
</tr>
<tr>
<td>Material Type</td>
</tr>
<tr>
<td>Idealisation</td>
</tr>
</tbody>
</table>

#### Finite Element Mesh Properties

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>Quadrilaterals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh Element</td>
<td>CCIsoQuad</td>
</tr>
<tr>
<td>Element Size</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>4.8675 m</td>
</tr>
</tbody>
</table>

The pier elements were thicker than the vault elements and the rest of the structure, and also modelled as plane strain, as the considerable depth of the elements support the entire structure. Therefore, the pier elements were modelled as such. The FE mesh here was slightly coarser than the other elements, due to the simple geometry of the pier elements.
5.3.4 Brick Masonry Parapet Material

The brick masonry parapet elements will also be modelled with the brick masonry vault material within all models, due to the same reasoning as with the spandrel wall and pier masonry material. Independent from the material properties, however, the stress state and FE mesh of the parapet was modelled separately to keep in line with the model assumptions, and those values independent from the vault masonry can be seen below in Table 5-4.

Table 5-4: Parapet Brick Masonry Material

<table>
<thead>
<tr>
<th>PARAPET MATERIAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Name</td>
<td>Parapet</td>
</tr>
<tr>
<td>Material Type</td>
<td>CC3DNonLinCementitious2</td>
</tr>
<tr>
<td>Idealisation</td>
<td>Plane Stress</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Finite Element Mesh Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh Type</td>
<td>Quadrilaterals</td>
</tr>
<tr>
<td>Mesh Element</td>
<td>CCIsoQuad</td>
</tr>
<tr>
<td>Element Size</td>
<td>0.25 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.5334 m</td>
</tr>
</tbody>
</table>

The parapet, being a mainly architectural element, contributes less to the actual structural behaviour than the other materials, so the finite element mesh was initially modelled coarser than the other material elements. Also, as with the pier masonry, it must be noted that the uncertainty of material parameters in historic structures limits the modelling process and engineering judgements need to be made, resulting in the use of the same parameters for multiple materials. The later stochastic analysis, however, will seek to compensate for these uncertainties.

5.3.5 Infill Material

The infill material is often the most uncertain material in historic masonry arch bridges, often consisting of gravels and soils found on or near to the site. (Gago et al., 2011) There was no existing data regarding the actual infill of the Rohrbach bridge at the time of model creation, so the infill material parameters had to be derived from previous tests and numerical simulations completed on similar structures. The material parameters adopted for the infill can be seen in Table 5-5.
The infill material was modelled with the Drucker-Prager failure criterion model, *CC3DDruckerPragerPlasticity*, as outlined in Section 5.1.3.2, and will utilize the plane strain state. The elastic modulus and Poisson’s ratio of the infill were adopted from the tests performed by Diaz et al., 2007 on a similar structure. The two Drucker-Prager parameters were calculated using the friction angle ($\varphi = 30^\circ$) and cohesion ($c = 0.025$ MPa) values also presented by Diaz et al., 2007. The weight density used was based on values adopted by Gago et al., 2011 and Galindo Diaz, Paredes Lopez, & Mora Mendez, 2007.

Due to the same geometrical restrictions of the spandrel wall elements in the initial models, the FE mesh elements used were triangular CCIsoQuad elements. The small angles of these regions resulted in meshing problems with quadrilateral elements; so triangular elements were used in the first models. The thickness of the infill elements was also based on half thickness of the entire infill material of the bridge.

### 5.3.6 Steel Rail Material

The purpose of modelling the steel rail of the train was to create a stiff element that was able to distribute the concentrated point loads from the train into the infill material. In reality, the train load is distributed to the infill through steel rails, timber sleepers, and a bed of large gravel. The effect of this dispersion of the concentrated train load is significant, but not modelled as such. Under the
assumption that the infill material elements are plane strain elements, the infill experiences equal
displacement along the element’s entire depth. The train load is, therefore, distributed and transferred
to the infill through the rail element without creating a punching phenomenon due to a highly
concentrated load. The material parameters adopted were taken from widely used values for steel
elements, and can be seen below in Table 5-6.

Table 5-6: Steel Rail Material

<table>
<thead>
<tr>
<th>RAIL MATERIAL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Name</td>
<td>Rail</td>
<td></td>
</tr>
<tr>
<td>Material Type</td>
<td>CCPlaneStressElastIsotropic</td>
<td></td>
</tr>
<tr>
<td>Idealisation</td>
<td>Plane Stress</td>
<td></td>
</tr>
<tr>
<td>Basic Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>E</td>
<td>2.10E+05</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>ν</td>
<td>0.3</td>
</tr>
<tr>
<td>Weight Density</td>
<td>p</td>
<td>7.85E-02</td>
</tr>
<tr>
<td>Finite Element Mesh Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mesh Type</td>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td>Mesh Element</td>
<td>CCIsoQuad</td>
<td></td>
</tr>
<tr>
<td>Element Size</td>
<td>0.1524</td>
<td>m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.2794</td>
<td>m</td>
</tr>
</tbody>
</table>

Due to the usage of this material mainly for load distribution purposes, a plane stress elastic isotropic
model was used to prevent nonlinearities from occurring in the element. This constitutive model, in
ATENA, is called CCPlaneStressElastIsotropic. The plane stress condition was used, as the rail is
considered to be a planar thin structure with regards to the infill and vault elements, and only vertical
forces were of interest. The same FE mesh properties as in the spandrel wall and infill elements was
used. The size and thickness of the rail material element was set to be equivalent to the cross-
sectional area of two typical steel rails.

5.4 Model Variants

5.4.1 Model 1

The first model created used the geometry and element layout observed in the original construction
documents of the bridge. This included the assumptions that the brick masonry vault was a constant
thickness throughout the depth of the bridge. Model 1 also did not include any haunching, as there
was no definitive data regarding the presence of haunching in the bridge. This model could therefore
be considered as a conservative model with regards to the displacements, as there were no additional elements within the structural arch to enhance the stiffness of the visible arch system. This model also served as the base of all later models. The different elements that comprised the model are shown below in Figure 5-14.

The model elements were each assigned the corresponding material and modelling properties outlined in Section 5.2. Based on these properties, the FE mesh was generated as seen below in Figure 5-15. The number of elements along the infill and arch interface lines were regulated in the mesh to force the most regular meshing possible.
The regulation of the meshing along the interface lines allowed for alignment of the nodes along the interfaces for successful kinematic linking between the elements. This regulation also insured that the arch masonry elements were aligned such that the interelemental interfaces were normal to the arch direction. This would best model the alignment of the individual bricks within the masonry and allow for more realistic crack formation.

5.4.2 Model 2

Model 2 was created by integrating haunching elements into the original FE Model 1. From the literature reviewed in Section 4, there was considerable evidence suggesting that bridges such as the Rohrbach bei Mattersburg bridge were likely to include haunching material connecting the vaults and easing the slope of the structural arch. Simplified haunching was created by observing the location of the infill drainage system, as seen on the original construction documents, and connecting that point to the arches. The bottom of the infill drainage pipe usually signifies the interface between infill and structural elements. The exact geometry of the model is attached in Annex 1, and the elements changed within the FE model to account for the haunching can be seen below in Figure 5-16.

![Figure 5-16: Geometric Revision for Model 2](image)

After the creation of the new Model 2 geometry, and further consultation with the ATENA developers at Cervenka Consulting, it was decided to update the FE meshing in Model 2. Due to changes in geometry easing the angles within the infill and spandrel wall elements, those elements were changed from triangular to quadrilateral elements. The quadrilateral elements within ATENA 2D were more recently developed and provide enhanced modelling capabilities compared to triangular elements. The newly formed haunching elements, modelled with brick masonry vault material, will continue using the triangular elements as they are still recommended for elements with difficult geometry. The two FE meshes, original and updated, can be seen below in Figure 5-17(a) and (b), respectively.
In addition to the modification of FE element shape in the spandrel wall, infill, and rail elements to quadrilaterals, a few small adjustments were made to the meshing size. The parapet mesh elements were reduced to the same size as that the spandrel wall, 0.1524 m, allowing for a more uniform interface. The rail elements were also reduced to 0.0520 m, to create a uniform quadrilateral mesh.

An additional alteration was made to Model 2, as it was concluded that this model with haunching would be the model to eventually be analysed in the stochastic analysis. Model 2 would, from this point forward, consist of two models Model 2A and Model 2B. Model 2A refers to the main model utilizing the updated mesh and the Drucker-Prager Plasticity constitutive model. Model 2B, on the other hand, will utilize a basic elastic isotropic constitutive model to determine if there is significant variation between the two models under service loading. The two models will be compared, along with the updated mesh validation, in a later section.

5.4.3 Model 3

Due to the nature of the infill material that was modelled and the material parameters assigned, the initial failure mechanism of Models 1 and 2 was later determined to occur within the infill. The development of shear bands and failure within the infill would cause failure and divergence within an FE model, but would not necessarily denote true catastrophic failure of the bridge. Realistically speaking, infill failure could cause the infill soil to shift and require attention to make the railway fully serviceable again, but would not cause the bridge to be a total, irreversible failure. This failure would have to occur within the actual structural elements of the bridge, namely the vault of the structure. This topic will be discussed in its entirety in Section 5.6.2.

For these reasons, Model 3 was created to remove the occurrence of infill failure by modelling the infill through a series of distributed loads representing the self-weight of the infill and the influence of the train loads distributed through the infill material were both modelled by distributed loads within separate load cases. The self-weight of the infill was calculated using the geometry of each half-span.
infill material element and its specific material weight. The calculated load was then distributed along the top interface of the arch and haunching elements considering the depth of the infill at each point, as seen below in Figure 5-18. This method was simplified by using trapezoidal distributions across the line elements of the interface.

Model 3 removed the second geometric body from the model; therefore, the procedure of kinematic linking was no longer necessary. With regards to the self-weight modelling in Model 3, modelling the influence of the train load through the infill element was a decidedly more complicated task. In order to model the influence of the train load, the reactions at the nodes along the arch extrados and haunching interface had to be monitored. Extracting the nodal reactions from the nodes along the interface after running an analysis with the full train loading did this. The analysis was a simplified analysis with an elastic infill material, as in Model 2B, and supports along the arch extrados interface removing chance for displacement so as to isolate the loading from the train. The nodal reactions for each node across the interface were then adjusted for the length of each element, so as to get the actual distributed reaction across the interface. The resulting distribution, seen in Figure 5-19, shows the reactions plotted versus the horizontal nodes along the interface from left to right.
Figure 5-19: Reaction pattern due to train loading

The two peaks in the graph represent where the point loads from the trains are located about midspan of the arch. This symmetric distribution was interpolated into trapezoidal distributed loads and transferred to the FE model. The final load distribution for the full train load on Model 3 is seen in Figure 5-20.

Figure 5-20: Train load influence on structural arch

The resulting model, Model 3, would therefore be used in order to test the structural elements of the bridge to failure without considering the failure of the infill. The isolation of the structural elements allowed for a more specific investigation into the mechanisms for a catastrophic failure within the bridge. These results would then be used to determine the safety margin for the bridge in terms of the current service loads.

In the determination of a safety margin for the bridge, however, Model 3 could not be used alone. Due to the nature of the 2D assumptions made at the beginning of the investigation, the spandrel wall in Model 3 contributed a higher amount of stiffness to the structural arch than in reality. The plane strain nature of the structural arch element assumes the displacement of the element is constant throughout its entire lateral thickness. This is an unrealistic assumption in terms of the 3D nature of the structure,
as the vault actually experiences varying displacement under train loading through the lateral dimension, as shown in the findings of Milia, 2012. Therefore, the displacement of Model 3’s vault assumes an average throughout the element, resulting in higher displacement at the keystone under the spandrel wall than in reality. This exposes the spandrel wall and parapet to higher stresses than in reality and could result in premature failure of the structure. Both Model 3 and Model 4 will then be developed to determine a proper safety margin for the bridge structure.

5.4.4 Model 4

Model 4 was created in order to obtain a conservative safety margin regarding the failure of the bridge’s substructure, as mentioned in Section 5.4.3. This was conducted by removing all influence of the spandrel wall on the structural stiffness by taking a 3-meter thick longitudinal section of the bridge. The thickness of the bridge section was determined by taking the width of the railbed and expanding outward from the rails in the lateral direction at an angle of 22°. This estimated a thickness roughly twice that of the distance between the rails, which is the industry standard 1.435 m. The model considered the conservative case that the arch receives no stiffness from the spandrel wall. This would then, in theory, result in a higher than realistic keystone displacement which could then be compared to the maximum displacements of the ATENA 3D models created by Milia, 2012. This model was created through adapting the geometry of Model 3 by replacing the parapet element with the steel rail element and replacing the spandrel wall material with that of the infill. The basic geometry and FE mesh of Model 4 can be seen below in Figure 5-21(a) and (b). All elements, excluding the rail element, were changed to a thickness of 3 meters.

![Model 4 Geometry](image-url)
The train loading from Model 2 would be reimplemented in Model 4, as point loads on the rail element. This would then be the final distinct model created for failure analysis. This model would be best suited for failure analysis of the vault structure. Removal of the stiffness attributed to the spandrel wall allowed for the observation of the behaviour of the vault under high loading and the failure mechanism would be visible. The failure loading of this model would be an underestimate, of course, due to the lack of influence from the spandrel walls. However, this would allow for a conservative bound on the safety margin for the bridge failure, and give insight into the mechanism with which the vault would fail.

5.5 Model Validation

5.5.1 Deformed Shape and Kinematic Linking

The first step in validation of the models was to observe the magnified deformed shape of the structure under both self-weight and full train service loading to ensure that the deformed shapes conformed to the expected patterns of the structure. The deformations were magnified 200 times in Figure 5-22, where it can be seen that the structure performed as expected with regards to the geometry and weights of the elements and the train point loads.

![Deformed model after self weight](image1)
![Deformed model after full train load](image2)

Figure 5-22: Model 2 Deformation Validation

The displacements were much higher in the infill element than the substructure, due to the assigned material properties, and the train load was dispersed through the rail element into the infill. The interface displacements between the kinematically linked elements displacement was also verified using monitoring points at corresponding nodes along the interface of the two elements. The resulting displacements were numerically identical, thus validating the successful kinematic linking between the infill and structural arch nodes.
5.5.2 Displacement Verification with 3D Models

In order to fully verify the 2D and 3D models being created concurrently through the current study and that of Milia, 2012, respectively, the results of the basic models were first compared. The results of the comparison were then validated with the results from the in-situ laser vibrometer tests conducted by Lechner, 2012. The validation of the models with the in-situ test results would therefore confirm the ability for the models to represent the structure and further analysis could continue.

Since the laser vibrometer test data that was obtained from Lechner, 2012 recorded displacements only at one point on the vault beneath the railbed, the results could not be directly compared to those of the 2D models developed in this study. The plane strain assumptions in the 2D models only returned an average lateral deflection of the vault element, and therefore could not account for the variance of deflections within the transversal direction. Thus, the 3D models of the bridge were first validated with the in-situ test results. (Milia, 2012) Upon validation, the 2D model average values could be compared to the lateral deflection variations within the 3D models. The self-weight deflection comparison graph is shown below in Figure 5-23. Models 1-3 were compared with three comparable 3D models to describe the lateral deflection behaviour due to the application of self-weight. Model B represents a model without haunching, comparable to Model 1. Model C represents a model with haunching and a Drucker-Prager plasticity model for the infill, comparable to Model 2A. Model D represents a model with haunching and an elastic isotropic model for the infill, comparable to Model 2B. Model 2 in this graph corresponds with Model 2A, as Model 2B was omitted from the graph due to its statistically insignificant difference from Model 2A under self-weight and service load conditions.

![Figure 5-23: Self Weight Keystone Deflection](image)

The self-weight deflection graph showed that the three comparable models set forth by Milia, 2012 follow similar trends as the 2D data. The addition of haunching reduced the deflection as compared to
without haunching, and there appeared to be no significant difference when modelling Drucker-Prager or elastic infill, at least with just the self-weight applied. Also, it could be seen that the 2D model averages were relatively lower than the 50th percentile of the 3D models, as would be expected with simple statistical analysis. The 2D model assumptions used created a stiffer structure than in reality, due to increased influence from the spandrel wall on the structure, and resulted in lower deflections. The 3D models, in principle, more realistically model the interaction between the spandrel wall and infill due to their physical proximity and the inherent assumptions in the models. The graph also shows Model 3 in comparison with the other models. This model, simplifying the infill load with distributed loads, resulted in slightly lower deflections than the other two models, due to the additional simplifications inherent in the model’s creation. The difference was small, however, and the model was used further for failure analysis.

After the self-weight deflection was compared, the full train service loading deflection was compared in Figure 5-24. This full train load graph shows the deflection due only to the application of the entire train load to both the 2D and 3D models.

![Figure 5-24: Full Train Load Keystone Deflection](image)

The 2D model averages here are higher relative to the statistical average of the 3D models compared to the self-weight results. This resulted from the fact that the adopted values for the entire representative load from each axle of the train were used in the 2D model. This was an overestimate, considering that the 2D models used represent one half of the thickness of the bridge, and should therefore be attributed one half of the loading under perfectly symmetrical conditions. The Rohrbach bei Mattersburg bridge is, however, eccentrically loaded with the rails being shifted to one side of the bridge rather than running along the central axis. Taking this into consideration, a plan was created of determining upper and lower bounds to create a plausible range of average deflection when restricted to 2D analysis. Therefore, the full train load would be the upper, conservative bound, considering that 100 percent of the train loading is transferred to one half of the bridge. This is unrealistic, in reality,
but provides a conservative upper bound for analysis. A lower bound was then also created considering perfectly symmetric conditions such that only one half of the train loading was transferred to one half of the bridge. The resulting graph is shown in Figure 5-25.

In this comparison, the lower bound was created to show the average displacement across the 2D model if only 50 percent of the service train load was transferred to one-half of the bridge. The real value of transferred load was unknown, but it was deduced that the eccentric loading transferred between 50-100 percent of the train loading to the most vulnerable half of the bridge. The vulnerable half was considered as the Southwestern half of the bridge, which receives more of the train loading and is where failure is likely to occur first. The range of values established between the upper and lower bounds is seen in Figure 5-26.
The range established showed that, in the 2D model of the bridge, the average deflection of the vault element varied vertically throughout the shaded green area of the graph. This represents the range of average deflections that could be expected when modelling under the 2D assumptions. For further confirmation, the 2D model average deflections were related to the average laser vibrometer reading of -0.34 mm deflection at the keystone. This value would then allow for an estimated range of average deflection expected within a 2D model as compared to the in-situ laser vibrometer test. The deflections and comparison values with the laser vibrometer test are seen below in Table 5-7.

Table 5-7: Average Keystone Deflections Under Train Loading

<table>
<thead>
<tr>
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<th>Full Train Load Applied</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 3</td>
</tr>
<tr>
<td>Average Deflection (mm)</td>
<td></td>
<td>-0.184</td>
<td>-0.178</td>
<td>-0.172</td>
</tr>
<tr>
<td>Percentage of laser vibrometer reading</td>
<td></td>
<td>54.00%</td>
<td>52.38%</td>
<td>50.68%</td>
</tr>
<tr>
<td></td>
<td>Half Train Load Applied</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>Model 1</td>
<td>Model 2</td>
<td>Model 3</td>
</tr>
<tr>
<td>Average Deflection (mm)</td>
<td></td>
<td>-0.092</td>
<td>-0.089</td>
<td>-0.086</td>
</tr>
<tr>
<td>Percentage of laser vibrometer reading</td>
<td></td>
<td>27.00%</td>
<td>26.19%</td>
<td>25.34%</td>
</tr>
</tbody>
</table>

It can be seen from the table that the lower bound is around 25% of the laser vibrometer reading, while the upper bound is around 54%. With this information, it was determined that the average...
keystone deflection of a 2D Model in this study with an asymmetrical train loading would be between 25 and 54% of the average value read directly beneath the railbed.

### 5.5.3 Updated FE Mesh Verification

After the FE mesh was updated for use in Models 2, 3, and 4, it was necessary to confirm that the results of the new meshing under service loads were consistent with that of the old mesh. In principle, the difference between the FE mesh, being mainly a change of element types, and not element sizes, would present a minimal change in behaviour. The resulting comparison between the old and new FE mesh in Model 2A can be seen in Table 5-8.

<table>
<thead>
<tr>
<th></th>
<th>Model 2</th>
<th>Model 2 New Mesh</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self Weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Keystone Displacement</td>
<td>mm</td>
<td>-0.391</td>
<td>-0.386</td>
</tr>
<tr>
<td>Full Train Load</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Keystone Displacement</td>
<td>mm</td>
<td>-0.569</td>
<td>-0.567</td>
</tr>
<tr>
<td>Keystone Displacement Only From Train Load</td>
<td>mm</td>
<td>-0.178</td>
<td>-0.181</td>
</tr>
</tbody>
</table>

As seen from the data, the difference between the two meshes varied less than 2 percent within the model for both the self-weight and train load displacements. This variation shows that the more refined mesh created slightly higher deflection under train service loading than the original mesh. This slight difference was most likely due to the varied distribution of the train loading through the different geometry of the updated elements.

### 5.6 Failure Analysis

Before the stochastic analysis was conducted, failure analysis was conducted on the various structural models to determine the failure mechanisms of the various elements. For this, it was necessary to determine the least favorable train loading location along the rails and conduct failure analysis on the bridge. This critical location, in this case, would consider where the train loading induces the most tensile stress and displacement within the structural arch. The determined location would then be used to test the failure of Models 3 and 4 to establish upper and lower bounds on the safety margin of the bridge.
5.6.1 Critical Train Load Location

To determine the critical train load location, a multi-step approach was taken to analyse the movement of the train load away from the center of the span at 0.5 m increments. Model 2 was used for this approach. The loads were shifted up to 2.0 m from center, and the model was exposed to full train service loads. The distribution of maximum principal stress $\sigma_1$, corresponding to the tensile stress within the elements was plotted on the model for each increment. These distributions can be seen Figure 5-27.
It was seen that the highest tensile stresses within the arch structure occur at the intrados of the arch around the keystone and around the interface of the arch extrados with the haunching element. From inspection during post-processing, the 1.0 m shift represented the most critical location. The
inspection was continued by comparing the principal stress $\sigma_1$, at the keystone node and surrounding nodes. Therefore, a graph was created to track the parameter as the train load was shifted from the center at the keystone node and the 4 adjacent nodes along the intrados to the right of the keystone. This graph is presented in Figure 5-28(a), along with the graph corresponding to the extrados interface nodes along the connection of the central arch extrados and right-hand side haunching material in Figure 5-28(b).

![Graph of Tensile Stresses in Keystone Area](image1)

![Graph of Tensile Stresses Near Arch/Haunching Interface](image2)

(a) Vault Intrados at Keystone  
(b) Haunching Connection at Vault Extrados

Figure 5-28: Principal stress $\sigma_1$ variation, in MPa, due to shift of train load

The keystone tensile stresses at the intrados midspan tended to be larger under the 1.0 m shift. The central node at the keystone experienced much higher tensile stresses than the others, as expected due to the arch geometry. The tensile stresses at the extrados and haunching connection, however, did not show as clear a rise at 1.0 m, but an increasing trend up to a 1.5 m shift. One last comparison, between the keystone displacements of each of the 5 models up to a single axle load of 0.3 MN (2.5 times full single axle service load), was conducted to further understand the response of the models. This graph can be seen in Figure 5-29.
The results showed that the keystone displacement was reduced with each shift further from the centered loading position. After considering all the data obtained and in accordance with widely accepted modelling rules of applying loads to shallow arched structures at a quarter-span for failure analysis, as mentioned in Brencich & Morbiducci, 2007, it was decided to apply the train loading with the 1.0 m shift from center. This loading pattern would then be used both for further failure analysis and for the stochastic analysis of the models.

5.6.2 Infill Shear Failure

Model 2 was the first model to undergo failure analysis. Using the adjusted train load location outlined in the previous section, the first failure mechanism observed in the models was located within the infill element of the bridge. The infill element developed shear bands following the paths where the highest concentration of the applied load travelled through the infill to the substructure. The first appearance of this shear band development, as seen in Figure 5-30, occurred with the application of a 0.53 MN single axle load, corresponding to 4.3 times the full train single axle load.
Plastic strains were visible at this point within the model, giving evidence of the start of nonlinear behaviour within the infill under the area of the train loading. The other elements within the model, however, had no nonlinearities. The analysis was continued, and the plastic strains developed along the entire path of the highest stress concentration through the infill to the interface of the arch structure up to a loading of 1.31 MN, as seen in Figure 5-31.

After 1.31 MN of applied load, or 10.7 times the full train single axle load, the FE model began to diverge. This was observed to be a result of the very high plastic strains in the infill leading to high deformations and distortions of the finite elements. It was seen that no cracks had opened within the structure, and the structural elements other than the infill all exhibited linear behaviour. This localized failure within the infill that led to divergence did not represent entire failure of the bridge or the substructure. Therefore, other models had to be utilized to test the actual capacity of the bridge's substructure.
5.6.3 Structural Element Failure

Due to the inability for Model 2 to properly model the failure of the structural elements leading to total failure of the structure, the other models created had to be utilized. These models removed certain elements of the structure in order to isolate the loadings most critical to observe the interaction of the train loading with the bridge substructure.

5.6.3.1 Model 3 Failure Mechanism

Model 3 was created using distributed loads to represent the infill and the influence of the train load through the infill on the bridge’s structural elements, and was next to be tested to failure. It must be noted that, due to time constraints within the realm of the project, the complex, centered loading pattern developed for the train load was used for this failure analysis. The 1.0 m shift, used in the other failure tests, was not able to be analysed in this failure test. The load displacement diagram shown in Figure 5-32 describes the behaviour of the FE model during the failure test.

![Model 3 Failure Test](image)

Figure 5-32: Model 3 Load Displacement Diagram

It can be seen that the behaviour within the model remains mostly linear until a significant crack opens through the parapet and spandrel wall elements directly above the right pier, as seen also in Figure 5-33(a). The crack does not signify complete failure, however, as the model was able to withstand additional loading after the crack’s appearance. The failure test was stopped well short of the peak, however, due to highly nonlinear behaviour caused by masonry crushing in the parapet element directly above the keystone. This behaviour, at the end of the test just before the model began to diverge, is seen in Figure 5-33(a) and (b).
The crushing of the parapet does not signify catastrophic failure in the substructure, as there only exists small tension cracking in the arch element, but the behaviour represented failure of the model due to divergence. Just before the model began to divergence, the bridge model had successfully been exposed to a single axle load of 1.78 MN, which corresponds to 14.5 times the full train single axle load. This behaviour would not be of overly high concern in the real bridge, as this unusually high stiffness could be due to the assumptions made in the model not to model the construction phases. Also, due to the rigid connections assumed between the elements and the plane strain state of the vault, there are more stresses transferred to the spandrel wall and parapet elements than in reality. Due to the results of this failure analysis, and that failure in the vault material had not yet been attained, a final failure analysis was conducted on Model 4.

5.6.3.2 Model 4 Failure Mechanism

The final failure test conducted involved Model 4, to observe the behaviour of the structural arch without the stiffness influence of the spandrel wall. The purpose of the failure test was to provide insight into the probable failure mechanism of the arch, and provide an approximate safety margin. Due to the lack of additional stiffness from the walls, the estimates in this test would yield conservative results. This, however, would allow for the attainment of a conservative safety margin, which could be the best possible margin to be extracted from the analysis. The failure analysis behaviour of Model 4 can be seen in Figure 5-34.
During the course of the test, it was not possible to obtain post-peak behaviour due to divergence of the FE model. This divergence was most likely attributed to the high plastic strains developing in the elements around the cracking pattern within the arch, or within the highly plastic area of the infill. Even after extensive adjustments to promote convergence into the post-peak behaviour, divergence was obtained. Due to time constraints within the present study, and the plateau reached in the load-displacement curve, the analysis was deemed complete. The results obtained showed a plateau in the load-displacement diagram at a single axle loading of 3.17 MN, corresponding to 25 times the full train single axle load. The behaviour leading to this plateau clearly shows the hinged failure mechanism developing within the arch, as seen in Figure 5-35.

The crack openings accentuated the tensile failure behaviour of the arch, and the deformed shape of the figure also suggested impending failure. Both tensile and compressive cracks could be seen in
the arch structure. From this cracking pattern and the monitoring of the arch deformation, a series of hinges was observed. The hinge development within the main arch of this model can be seen outlined in Figure 5-36.

![Figure 5-36: Failure Mechanism and Crack Width [m] at Single Axle Load of 3.17 MN](image)

The failure test on Model 4 allowed conclusions to be drawn on the probable failure of the actual arch mechanism. The conservative failure model results showed that the brick masonry vault of the Rohrbach bridge would fail under a train load of approximately 25 times that of the full train service load. The isolation of the vault element from the additional stiffness added by the spandrel walls in this model allowed for the determination of the actual global failure mechanism of the vault. As the vault would be the critical element within the structure, and that its failure would cause catastrophic failure of the structure, the results from this last failure analysis allowed for a conservative and confident safety margin of the capacity of the entire structure. With the failure analysis completed, the study was then shifted to the stochastic analysis of the bridge.
6. STOCHASTIC ANALYSIS

The use of stochastic analysis in the scope of this project took the FE models from the deterministic analysis even further toward realistic modelling of the materials of the bridge. In standard FE analyses, each material within a macroelement represents a homogeneous continuum with regards to its material properties. The material properties, input by the user during material creation, are therefore equal at every point within the designated macroelement. This assumption is typically acceptable within more continuous materials as concrete, but becomes less valid when used to model historic masonry or infill materials. Before construction regulations were set in place, the inhomogeneities within materials were higher. Only later did it become typical construction practice to standardize and document materials and methods. Also, the vast difference in material properties between bricks, stones, and mortar in masonry present a mesh of varying properties throughout the complex material. The usage of random fields within a stochastic analysis would attempt to mimic the distribution of this variation, and the inhomogeneities within the materials of the bridge. This would serve as a reliability analysis, which could be used to better approximate the safety of the bridge.

6.1 Goals and Principles

With the goal of being able to more accurately model the inhomogeneities of material properties, the models will be run several times with various analysis goals in order to determine the effect of the inhomogeneities on the structural behaviour. The initial analysis will be attempted with simultaneously randomizing all bridge element materials. In the case that the processing limitations restrict this analysis, each element will be randomized separately, and the results synthesized to determine the effect of the variability within each element on the overall performance of the structure.

Using the SARA software studio, several samples of random fields were used to attempt to observe the behaviour of the bridge structure when considering the inhomogeneities. The basic procedure of the SARA Studio usage will be presented next in Section 6.2, while the input parameters and corresponding analyses will be presented in the following sections.

6.2 SARA Studio

The SARA (Structural Analysis and Reliability Assessment) software package integrates the FE program ATENA with the reliability software FREET. The combination of these two softwares allows for a nonlinear probabilistic and reliability analysis of ATENA FE models.

The SARA studio provides management between the two programs and allows for a more seamless project analysis. (Havlasek & Pukl, 2012) Individual projects are created and all files organized through the graphic shell, so that each separate analysis can be conducted and filed accordingly.
6.2.1 Analysis Preparation

The first step was to develop the deterministic FE model within ATENA, with all appropriate load steps and monitoring points for the stochastic analysis already prepared. New materials, based on the materials from the deterministic analysis, had to be assigned to allow SARA to recognize which materials were to adopt the random fields input. The input file was then uploaded into the SARA, where a new file directory was created. The material and model parameters to be randomized could then be selected, and the method of randomization could be defined. The randomization was either via random variable or random field input. After the parameters were selected, FREET could be started and the actual stochastic variation parameters set to define the random fields. Once the fields were defined, FREET uses the Latin hypercube sampling method to generate the actual random fields for each variable and material requested. The random fields were then visible to verify within the FREET interface. Once the fields were produced, the program created ATENA input files for each of the simulations, and the program could then run the FE analyses through ATENA for each of the simulations. Once the FE analyses were completed, the FREET interface can be used to produce histograms and graphs of the structural behaviour and sensitivity analysis.

6.2.2 Random Fields

The typical construction of a finite element model creates homogeneous material elements where the material input parameters are constant throughout the entire continuum. This is only an idealization of a material and does not realistically represent the distribution of parameters throughout a material. This is especially evident when modelling masonry structures. The typical homogeneous material element within the model does not account for the change in parameters between the individual brick and mortar elements within the masonry, nor the fact that the distribution of strength or density parameters throughout those smaller elements is not homogeneous. This is where the usage of random fields comes into the procedure. A random field is basically a list of values that can be thought of as a heterogeneous spatial field that has attributed random values to each point within the spatial continuum. (Ostoja-Starzewski, 1998) For the sake of brevity, the mathematical details behind the random fields concept will be left out of this report. For more information on the meaning and theory behind random fields, please consult the FREET Theory manual by Novak et al., 2002 or the works of Ostoja-Starzewski, 1998.

In the case of a structural model, this would be random values of a predetermined parameter throughout the continuum according to a predefined distribution. The variation of the parameters is described in Section 6.5, and the generation of the random fields is described in explicit detail in Section 6.6.1.
6.3 Variation Of Parameters

In order to model the randomization of the different material parameters within the structure, the values adopted in the previous section were used as mean values for the random value distribution. Coefficients of variation for each parameter were then found using the same procedure as with the original material parameters for the FE Models. Data from in-situ tests on the Rohrbach bei Mattersburg bridge was used where available and the remaining values were obtained from similar tests conducted on similar structures and materials. The values adopted through this procedure for the materials in this study can be seen below in Table 6-1.

Table 6-1: Initial Variation of Parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>Variables</th>
<th>Symbol</th>
<th>Adopted Value</th>
<th>Units</th>
<th>COV (%)</th>
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<tr>
<td>Brick masonry</td>
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<td>E</td>
<td>2083.00</td>
<td>MPa</td>
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<td>$f_c$</td>
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<td></td>
<td>Tensile fracture energy</td>
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<td>N/m</td>
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</tbody>
</table>

The initial values obtained for the variation of parameters was relatively high, both from the experimental data and data obtained from other studies. Brick masonry compressive strength variation and weight density were obtained from statistical analysis of the core sample test results. (Eisenhut, 2012) The variation of the tensile strength and fracture energy within the brick masonry was also obtained from values adopted by Schueremans & Van Gemert, 2005. The variation in properties in both brick and stone masonry in terms of elastic modulus, compressive strength, and tensile strength were found to be similar in many sources, such as in Verstrynge et al., 2012, so similar values were adopted. Elastic modulus variation parameters were adopted from values obtained by tests of Verstrynge et al., 2012. The variation of infill parameters was obtained from the higher
bound of the range most recently published by the Joint Committee on Structure Safety guidelines for soils. (Baker & Calle, 2006)

The variation parameters observed were quite high, as the historical materials and lack of precise material test data resulted in a great deal of uncertainty. The current limitations on the FREEET software within SARA studio require the usage of normal distributions when producing values within random fields. This limitation presented concerns that the random fields could include negative values of parameters that could result in FE analysis problems and failure of the analysis. Therefore, validation of the parameter variation had to be conducted.

### 6.4 Validation of Parameter Variation

In order to validate the parameter variation and to ensure low probability of obtaining negative values, a statistical analysis was conducted using stochastic simulations within the mathematical software, MATLAB. (MATLAB, 2011). An algorithm was written to create normal distributions of random values dependent upon a mean value and coefficient of variation. The parameter values and coefficients of variation from Table 6-1 were used for these simulations. For each parameter, 5 separate sample tests were run, each with 1000 random values. This was to obtain a normal distribution curve and observe the probability of obtaining only positive values within those curves with our chosen values. The criteria defined for this test was that the probability of positive (or usable) values should be greater than or equal to a variation of 3 standard deviations from the mean, or 99.7 percent. Examples of the graphical distribution plot results of these tests can be seen in Figure 6-1.

![Sample Distribution Plots from Validation Procedure (MATLAB)](image)

(a) Brick Masonry Compressive Strength  
(b) Brick Masonry Tensile Fracture Energy

Figure 6-1: Sample Distribution Plots from Validation Procedure (MATLAB)

The sample distributions for parameters such as the brick masonry compressive strength and tensile fracture energy show a small portion of the curve crossing into negative values. This would lead to problems within the FE analysis, and was to be avoided. Therefore, the coefficients of variation were
lowered in each of these parameter distributions until a probability equal to or greater than 99.7 percent was obtained. This did not remove the negative values from the distribution, rather lessoned the probability of obtaining these values and a premature FE analysis failure. The adjusted variations, after this procedure, are shown in Table 6-2.

Table 6-2: Adjusted Variation Parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>Variables</th>
<th>Symbol</th>
<th>COV (%)</th>
<th>Probability of Usable Values After Statistical Analysis</th>
<th>Adjusted COV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick masonry</td>
<td>Elastic modulus</td>
<td>E</td>
<td>30</td>
<td>0.9996</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Compressive strength</td>
<td>f&lt;sub&gt;c&lt;/sub&gt;</td>
<td>51</td>
<td>0.9762</td>
<td>0.9970</td>
</tr>
<tr>
<td></td>
<td>Tensile strength</td>
<td>f&lt;sub&gt;t&lt;/sub&gt;</td>
<td>51</td>
<td>0.9777</td>
<td>0.9970</td>
</tr>
<tr>
<td></td>
<td>Tensile fracture energy</td>
<td>G&lt;sub&gt;f&lt;/sub&gt;</td>
<td>50</td>
<td>0.9786</td>
<td>0.9974</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>ρ</td>
<td>5</td>
<td>1.0000</td>
<td>--</td>
</tr>
<tr>
<td>Stone masonry</td>
<td>Elastic modulus</td>
<td>E</td>
<td>30</td>
<td>0.9995</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Compressive strength</td>
<td>f&lt;sub&gt;c&lt;/sub&gt;</td>
<td>51</td>
<td>0.9995</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Tensile strength</td>
<td>f&lt;sub&gt;t&lt;/sub&gt;</td>
<td>51</td>
<td>0.9779</td>
<td>0.9971</td>
</tr>
<tr>
<td></td>
<td>Tensile fracture energy</td>
<td>G&lt;sub&gt;f&lt;/sub&gt;</td>
<td>50</td>
<td>0.9786</td>
<td>0.9974</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>ρ</td>
<td>5</td>
<td>1.0000</td>
<td>--</td>
</tr>
<tr>
<td>Infill</td>
<td>Elastic modulus</td>
<td>E</td>
<td>10</td>
<td>1.0000</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>ρ</td>
<td>10</td>
<td>1.0000</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Drucker-Prager alpha</td>
<td>α&lt;sub&gt;DP&lt;/sub&gt;</td>
<td>20</td>
<td>1.0000</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Drucker-Prager k</td>
<td>k</td>
<td>50</td>
<td>0.9773</td>
<td>0.9979</td>
</tr>
</tbody>
</table>

The value at left within the probability columns show the initial tests with the COV values obtained through the procedure outlined in Section 6.3. Many of the values obtained acceptable (green) results, meaning greater than 99.7 percent acceptable values. The other parameters, however, had to be adjusted until the criteria were reached. The new coefficients of variation were then adopted for use in the stochastic analysis.

6.5 Analysis Procedure

Once the adjusted values were set, the values were ready to be input into FREET for random field generation. The analyses to be conducted were first a service load test and then a failure test on Model 4. It was concluded from the deterministic analysis in Chapter 5 that Model 2, which included the spandrel wall elements in addition to the infill, presented unrealistic results. The influence of the spandrel wall was too high, resulting in an overly stiff structure that was unable to reach the critical
state of the substructure failure. Therefore, the stochastic analysis results conducted in following sections refer only to the tests conducted on Model 4, which was the model able to most properly model the critical behaviour of the structure.

This procedure set to first determine the effect of the heterogeneity of the materials under the actual expected service loads and compare keystone deflection data with the deterministic analysis results. Later, multiple random field models were analysed toward failure to observe the appearance and type of failure mechanisms. The results from this analysis would allow for a better understanding of the behaviour and probable ultimate state of the bridge, and allow for a better approximation of a safety margin for the structure.

Due to the limitations in the distribution settings within FREET for random field generation, difficulties could arise when randomizing materials with high uncertainties. Even after setting a three standard deviation criteria and lowering the coefficients of variation as discussed in Section 6.4, several models yielded unacceptable values within the random fields. Several variants were used until an acceptable set of variation parameters yielded working FE models. The variation parameters can be seen in Table 6-3. Additionally, the original goal of this procedure was to use the detailed final results of the GPR testing to properly calibrate the correlation lengths within the random fields to produce as accurate heterogeneous parameter distributions as possible. However, at the time of analysis, it was not possible to obtain a final report from the GPR analysts, as it was still being completed and was not available for release. The data available was mainly qualitative data that gave approximations of interface depths and overall images of the distribution, but not precise enough to use for correlation length calibration. Also, the lack of opportunity for a site visit prevented exact measurements of the material geometries. Thus, judgements had to be made to apply correlation lengths within the materials. The masonry material correlation lengths were set to approximate the typical size of the brick and stone elements. The brick elements utilized 0.3 m and 0.1 m correlation lengths in the horizontal (dx) and vertical (dy) directions, respectively. The stone and infill elements were then set to have 0.4 m correlation lengths in both directions, while the infill values were set much smaller, to have 0.1 m correlation lengths in both directions. These values can be seen in Table 6-3.
The values in Table 6-3 represent the updated values used for the generation of the random distributions, or fields, within the stochastic analysis. The Latin hypercube sampling method was used to create normal distributions of each of the parameters listed, with the only exception being weight density, $\rho$, which will be discussed later in this section. The adopted values for each parameter from Table 6-1 were set as the mean values of the distributions, while the standard deviations were calculated from the coefficient of variation (COV) values set forth in Table 6-3. Further development of the software should include the option for log-normal distributions for random field generation, to prevent the requirement of lower COV values (marked red in Table 6-3), as needed in this analysis. The mentioned limitations and sampling complications restricted COV values for the masonry elements to 25 percent. The COV values for the infill parameters, however, did not present complications within the sampling and were able to remain

Due to the time constraints within the study, and the amount of processing power needed for random field generation and the corresponding FE analyses, the stochastic analyses were limited to 6 random field model simulations. The service load analysis utilized 6 separate models. Each of these models included a unique random field for each of the randomized parameters. The failure analysis then generated 6 additional model simulations, each with their own unique random fields. This would allow for a substantial test set and yield results for consideration. However, larger sample sets and further analysis would benefit the study and yield more certain and more practical results.
It must be noted that due to the recommendations and limitations within the FREET software, the weight density was not randomized within the stochastic analysis. Since the weight density variation was also small relative to the other parameter variations, this removal of the randomization of this parameter was expected to have less effect than the other parameters. This slight simplification in the procedure also resulted in quicker processing times for both the random field generation and model analysis.

6.6 Results

Results in this section were obtained through tests conducted on Model 4 discussed in the earlier chapters. This model, representing a 3 m thick cut of the interior of the bridge was used for the creation of the stochastic distributions and model analysis. Due to the fact that this model was able to produce conservative results while also allowing for the observation of the substructure failure, this model was judged to be the best suited for stochastic analysis.

6.6.1 Random Field Generation

After the Latin hypercube sampling process was completed, random fields were created for each parameter randomized within each material. The random fields were visualized within the FREET interface before they were input into the ATENA files, so that verification of the distributions could be made. The random fields of the brick masonry material for one simulation in the service load test are shown in Figure 6-2.
In addition to the parameters randomized in the masonry materials, the infill material's Drucker-Prager coefficients were randomized. The corresponding random fields for the Drucker-Prager specific parameters, shown within FREET, can be seen in Figure 6-3.

These visualizations in FREET allowed for the validation of the random fields generation before being input into ATENA. Once validated and input into the ATENA files, the random field files were accessed through a reference in the FE analysis input file, so that the heterogeneity of the materials could be modelled. Each of the random fields within the materials could be visualized within the postprocessing interface within ATENA. This visualization is much more of a visual aid, as it allows for the visualisation of the material parameter distribution directly within the material.

Random fields were selected for each material within one simulation for visualization. These visualizations, created within the ATENA postprocessing interface, show one of the networks of
random fields that were included within one of the six failure models. The elastic modulus random fields created for the three materials, stone masonry, brick masonry, and infill, are shown in Figure 6-4.

![Figure 6-4: Random Fields for Elastic Modulus, E [MPa]](image)

Two visualizations had to be made for the elastic modulus, because the values required different scales to properly show the distributions within the different materials. The differences in correlation lengths discussed in Section 6.5 are visible, as the spatial variability in the infill varies much more quickly throughout the model than the masonry materials. The fields of compressive strength within the stone and brick masonry are seen in Figure 6-5.
The compressive strength variation also shows the variability due to the difference in correlation length between the two types of masonry. The brick masonry was designed to have a smaller correlation length, to model the fact that the bricks in brick masonry are much smaller than the stones in the stone pier masonry, therefore the variation within the macroelement of the masonry occurs faster. In addition to the compression strength, the tensile strength was randomized, as seen in Figure 6-6.

Although the randomization inputs for the infill material did not include tensile strength, randomization appears here because of the algorithm in the post-processor. The $a_{DP}$ coefficient parameter was the second randomized parameter within the infill, while the tensile strength was the second parameter randomized in the masonry. This resulted in simultaneous visualization of the two parameters. The last set of random fields observed within the masonry elements were for the tensile fracture energy. These fields are seen in Figure 6-7.
The parameters within the infill were also randomized, and the effect of the randomization could dictate how the train loads are distributed to the masonry elements. The random fields for the two coefficients, $\alpha_{DP}$ and $k$, are seen in Figure 6-8(a) and (b), respectively. These coefficients define the shape of the yield surface and their variation could lead to weakened areas within the infill.

(a) Random Field for $\alpha_{DP}$ coefficient

(b) Random Field for $k$ coefficient

Figure 6-8: Random Fields for Drucker-Prager Coefficients
The initial procedure plan for the random field generation in both the masonry and infill elements included the comparison of the random fields distribution with GPR results from the tests conducted on the structure for validation. However, the full report of the GPR results was not completed during the time window needed and was unavailable to use for the validation in this study. Only a preliminary set of results was available, and a rough qualitative comparison of the variations between the compressive strength and the GPR results is seen in Figure 6-9.

![Figure 6-9: Variation Validation with GPR Results](image)

It is difficult to fully compare the two results, because of the lack of proper scale and view. However, the pattern of variation seen in the brick masonry material elements in Figure 6-9(a), due to the chosen correlation lengths provides a similar distribution of inhomogeneities as seen in the GPR figures created from a test within the arch in Figure 6-9(b). Further study into this correlation with additional, and more quantitative GPR results would be integral to fully successful validation of the random fields generation.

### 6.6.2 Service Load Analysis

The service load tests were conducted on Model 4 up to the full train loading that was used in the deterministic analysis. These tests utilized the centered train loading that was considered during the early tests of Chapter 5, so that the results could be compared to the results found in the deterministic analysis. After 6 simulations of random fields were created, the FE models were subjected to their own self-weight, and compared in Figure 6-10.
Each of the models is seen to exhibit only linear behaviour when the self-weight was applied. No cracking occurred during this loading. The resulting variation within the 6 random field simulations showed a mean value of 1.81 mm deflection. This was 2 percent higher than the deterministic model, showing 1.77 mm of displacement. The comparison with the deterministic model behaviour is also seen in Figure 6-11, along with the mean and standard deviation of the distribution. The coefficient of variation within the 6 simulations was 5 percent after the entire self-weight was applied.
It can be seen that the mean displacement is higher than that of the deterministic model. The deterministic model still lies within one standard deviation of that mean, however, so it is seen that the variance within behaviour is not substantially large due only to the randomized materials under self-weight application.

The 6 models were then tested to the full train loading. The resulting displacements were adjusted to remove the self-weight displacement. This isolated the displacement due only to the train loading. The results obtained are compared to the deterministic model, and shown in Figure 6-12.

![Figure 6-12: Keystone Displacement Under Service Loading](image)

The models continued to exhibit only linear behaviour under service loading. Miniscule plastic strains had appeared at random locations within the infill, causing slight concern that small, localized slip within the infill could occur under service loading and loading slightly greater. This however, would not present a significant effect to the structural behaviour. The resulting distribution resulted in a mean value 2.4 percent higher than the deterministic model, as shown in Figure 6-13. The coefficient of variation of the 6 models was 4 percent, slightly lower than the variation in the self-weight deflection.
The resulting distribution showed higher deflections than the deterministic model, as expected. Also, the deterministic model did not fall within one standard deviation from the mean of the random field distribution. This showed substantial evidence that the heterogeneity of the materials resulted in larger displacements under train loading. The heterogeneity of the materials includes weaker properties in certain areas. The fact that there is variation throughout the materials makes them inherently weaker due to lack of continuity and, therefore, the displacements of the keystone appeared to be higher after the application of random fields.

**6.6.3 Failure State Analysis**

After the completion of the service loading tests, 6 additional randomized simulations were created and subjected to train loadings shifted 1 m to the right. This position was judged to be the most unfavorable to the bridge, as in Section 5.6.1. The 6 models were then subjected to failure analyses. Each of the models was brought to failure independently.

**6.6.3.1 Failure Curve Comparison**

As with the deterministic model, the FE analyses did not reach post-peak behaviour in any of the 6 analyses. Each of the models failed prematurely due to highly nonlinear behaviour that resulted in eventual divergence of the numerical model, even after careful consideration and adjustment of solution parameters. The results of the failure analyses are presented in Figure 6-14.
As seen in the load-displacement graph, there was a substantial level of variation within the nonlinear behaviour in the 6 models, as compared with the failure curve produced from the deterministic model. The highest single axle train load reached before failure within the 6 models was 3.427 MN (28 x full train load), while the lowest was 2.1721 MN (17.7 x full train load). The comparison between different important factors observed throughout the failure analysis can be seen in Table 6-4.

The comparison between the random field simulations and the deterministic model shows that 5 out of 6 of the randomized models developed a connected shear band through the infill element, observed through plastic strains. Also, 4 out of 6 randomized models also developed cracks earlier than the deterministic model.
deterministic model. These results showed that the variation of the parameters throughout the materials appears to result in a weaker structure.

While the original deterministic model was created to be a conservative model, the results obtained from the stochastic analysis showed the deterministic model to be less conservative than the majority of the models accounting for the spatial variability within the materials. This finding was integral in the understanding of the structural behaviour, as the heterogeneous models were able to more accurately reproduce the parameter distribution within the heterogeneous masonry and infill materials.

6.6.3.2 Individual Random Fields Model Failure Mechanisms

Each of the 6 models tested to failure were analysed at the failure state to understand how and why each model failed. There were three typical mechanisms for failure of the numerical models, similar to mechanisms observed in the deterministic analysis. The first resulted of high plastic strains created due to the shear band formation through the infill material due to the high train loads. The second mechanism resulted from the highly plastic behaviour due to crushing of the masonry arch in compression. The third was the failure of the structural arch in tension. The figures in this section visualizing the failure mechanisms are shown with a 5 x magnification of the deformed shapes of the structure for better visualization of the structural behaviour.

The first random field model failed at a single axle load of 2.59 MN, corresponding to 21 times that of a full train load. This numerical model failure was attributed to the emergence of high plastic strains in the infill representing the shear band formation. The model at failure can be seen in Figure 6-15.

![Figure 6-15: Equivalent Plastic Strains in Model RF 1](image)

These shear bands are typical when using a highly concentrated load. However, the shear bands in this model were very high in magnitude. Because these bands typically represent a movement of the soil particles within the infill, the $\alpha_{DP}$ coefficient, which controls the shape of the Drucker-Prager yield
surface of the material, could have an effect on the separation or movement of the infill particles. Therefore, the $\alpha_{DP}$ coefficient random field was analysed, as seen in Figure 6-16.

![Figure 6-16: Drucker-Prager $\alpha_{DP}$ Distribution in Model RF 1 [MPa]](image1)

Due to the algorithm within the post-processor, the tensile strength of the masonry was also included in this visualization. The random fields show a cluster of lower values of $\alpha_{DP}$ in the area of the shear band shown in Figure 6-15. This meant that lower yield stresses were present in this area of the material beneath the right train load where the shear band had formed. This high concentration of low yield stresses allowed for the plastic strains within the model to propagate more quickly, leading to the eventual divergence of the numerical model.

In the second random fields model, RF 2, the failure of the numerical model was seen to have occurred not only through excessive plastic strains within the infill, but due to the emergence of plastic strains within the compression areas of the structural arch. These areas, circled within Figure 6-17, experienced extensive compression cracking. The emergence of plastic strain in these areas due to this compression cracking led to the failure of the numerical model.

![Figure 6-17: Minimum (Compressive) Principal Stresses of Model RF 2 [MPa]](image2)

This phenomenon was not present in all models, as several models failed before the high loading (3.36 MN single axle load) present in model RF 2. This loading represented 27.4 times that of the full...
train load. Due to this phenomenon, the random fields for the compression strength of the masonry were consulted for this model to observe the variation. It was found that the distribution presented weak points in compressive strength, circled in Figure 6-18, at the critical locations of the arch that could result in premature compression failure.

![Figure 6-18: Compressive Strength Distribution in Model RF 2 [MPa]](image)

The compressive crushing phenomenon was likely accelerated due to the lower compressive strengths in these areas. This correlation shows that the variability of materials such as this historic masonry present problems in that weak points can occur often throughout the materials.

The third random fields model, RF 3, exhibited the third failure mechanism, due to tensile failure, in the structural arch at a single axle train load of 3.427 MN. This corresponded to 28 times the full train load, and was the highest load experienced by any of the randomized models. The tensile failure mechanism was discovered as the elements in the area of the large cracks began to deform greatly and the plastic strains became very high, leading to the divergence of the model. The cracking pattern most responsible for the failure is seen in Figure 6-19.

![Figure 6-19: Crack Width in Model RF 3 [m]](image)
The cracking of the masonry material was due to tensile failure within the element, which experienced high local distortion corresponding to a tensile blowout of the material element. Therefore, the random field for tensile strength was consulted to establish a connection between spatial variability and the tensile failure of the arch. This tensile strength distribution is seen in Figure 6-20.

![Figure 6-20: Tensile Strength Distribution in Model RF 3 [MPa]](image)

The circled area, representing the area of the tensile failure exhibited low tensile strength comparing to the rest of the brick masonry element. This low strength area combined with the high tensile stress in the area led to the tensile failure mechanism.

The remaining three models failed with one or more of the three failure mechanisms presented here. The models failed at single axle applied train loads of 2.42 MN, 2.17 MN, and 3.02 MN, corresponding to 19.7, 17.7, and 24.6 times the full train load, respectively. Therefore, the range of divergence failures of the models were between 17.7 and 28 times the full train load. These results, seen previously in Table 6-4, combined with the deterministic model failure of 25 times the full train load establish a conservative range of failure loads. Since the FE models with the lowest failure loads failed due to plastic strains in the infill, which was determined to not present a true failure of the structure, their values were not fully considered when adjusting the safety factor. The average failure load of the randomized models, however, was still slightly lower than that of the deterministic model, so the deterministic safety margin presented in Section 5.6.3 was lowered. The updated, conservative safety margin for the Rohrbach bei Mattersburg bridge is approximately 20 times the full train loading. This safety margin, based on the assumptions made in this study, shows that the structure was very conservatively built, and that current service loads will not present danger to the structural elements of the bridge.
7. CONCLUSIONS

The present study provided a logical and tested procedure for the analysis of uncertain structures. The scientific method developed within this study worked through the various stages of analysis on the Rohrbach bei Mattersburg bridge case study to demonstrate an analysis method based on two-dimensional finite element models. The study sought to determine how to analyse a historical structure with very little remaining historical information and limited knowledge of materials or the interior composition of the structure. The models were then validated with experimental results through collaboration with the development of corresponding 3D models.

Engineering judgement became very important in this study, as usual in analysis of historical structures due to the general uncertainty of the structure, lack of information, and inability to conduct additional in-situ tests. Due to the geographic location of the bridge, time constraints within the project, and restrictions on the visitation of the bridge, which is still in service, neither a site visit nor inspection was conducted on the bridge. Therefore, judgement had to be best used to interpret the data from the limited geometrical data and the results of previous in-situ tests. Also, from research deeper into similar structures and interpretation of the GPR results, qualified assumptions were made into the interior composition of the structural vault.

The complex nature of the finite element models including infill tested the capabilities of the ATENA software to run multiple geometric bodies within one model, while manually editing the input file to kinematically bond the two bodies together. This procedure essentially allowed for the creation and observation of more complex, conceptually three-dimensional models, within a two-dimensional analysis program. The kinematic linking procedure was successful in connecting the elements between the two bodies when considering their plane strain state assumptions. The behaviour at the connected interface was monitored, as well as points throughout the materials in order to confirm the successful integration of the two bodies.

The eccentricity of train loading on the bridge added complexity to the problem. This eccentricity would typically require a 3D model considering the exact location of the rails. Due to the 2D limitations, the exact transversal location could not be used. Also, the plane strain assumption established an average displacement through the depth of the bridge and would therefore not be affected by the change in location of the rails. Therefore, an upper and lower bound was established to create a range of expected deflection due to the eccentric train loading. This was an approximation to relate the experimental keystone deflection under the railbed to the 2D model keystone deflection.

The deterministic analysis of the structure further developed and executed a protocol that yielded an array of results regarding the interaction of the different elements in the structure. The analysis was able to predict the failure mechanisms of the infill material, spandrel wall material, and the structural vault using specialized models to isolate the behaviour of interest. These models included homogenized material elements using material parameters derived from in-situ tests and previous tests on similar structures. The deterministic homogenized models allowed for the creation of a safety
margin of 25 times the full train load before substantial failure of the bridge’s substructure. This safety margin, however, is a conservative value due to the removal of the stiffness influence from the spandrel walls.

The deterministic results presented in this study should be combined with additional 2D models along the transversal axis to observe the longitudinal cracking observed through the 3D numerical model study by Milia, 2012. The failure patterns observed in the 3D numerical models showed the importance of cracking in the longitudinal direction in the failure mechanisms of the bridge. These cracks, however, did not produce failure before the loading limit presented in this study, due to the additional stiffness added by the contact interaction between the spandrel wall and the infill materials. The 3D models produced by Milia, 2012, however, could not consider the spatial variation of the material parameters, and stochastic analysis was conducted only via the random variables method. Further study and collaboration between 2D and 3D analyses would provide a more precise safety margin and failure analysis.

The failure analysis of the random fields models allowed for investigation into the correlation between the spatial variability of certain parameters and the behaviour and ultimate failure mechanism of the structure. It was discovered that, if the spatial variability of the material parameters was known, the heterogeneity and weak areas within the material could greatly influence the location and type of failure within the structure. There was an observed relationship between areas weak in compression with failure due to compressive crushing of masonry. A relationship was also observed between areas weaker in tension and tensile failure due to cracking. Within the infill, the definition of the yield criterion throughout the structure also yielded failure in weak areas. These results were of significant importance to support the usage of spatial variability in structural modelling of masonry. The modelling of masonry as a homogeneous material in numerical models, even after adjusting values to account for brick and mortar element properties, cannot fully capture the actual nature of the material. This is especially true in historic structures, where a heterogeneous material such as brick masonry is exposed to the elements for several years, or hundreds of years. Each location within the greater medium has been exposed to different phenomena throughout its lifetime and these phenomena, including weather, physical attack, or freeze/thaw cycles, can ultimately change the material parameters. Under these conditions, and from the results presented in this study, it appeared that modelling these materials as homogeneous continua could not create a truly realistic estimation of the behaviour of the structure.

The present study was able to contribute to the state of the art of structural analysis of historic structures by using a regimented, logical protocol to observe structural behaviour. This protocol was however, inherently limited due in fact to the requirement of using 2D modelling software. Nevertheless, the creation of multiple models, along with collaboration and validation with 3D models in a concurrent study, allowed for a higher level of confidence in the results of the 2D analyses. The advanced model creation involving the integration of multiple kinematically linked 2D bodies enabled
more advanced observation of each material's behaviour under different types of loading. This innovative procedure could be utilized in further studies to advance the current state of analysis procedures for masonry arch bridges. The comparison between deterministic and stochastic analyses presented results that show that the spatial variability within the materials is important to consider when modelling. The parameter variation presented weaker behaviour and showed different possible failure mechanisms based on the distribution of the variation.

It is recommended that further study in this direction be conducted, as the state of the art and the software is developed further. The stochastic results were limited due to time constraints, the software limitations considering random fields of high variation, and the implementation of random fields into 3D models. With the further development of this type of analysis and the stochastic software, however, a more extensive stochastic study could be conducted, more simulations could be processed, and the topic could be further validated. Also, if more quantitative ground penetrating radar data were available, a more precise validation procedure could be conducted. Nevertheless, this study contributed to greatly to the understanding of the global behaviour of the Rohrbach bei Mattersburg bridge considering the high level of uncertainty within the structure.
8. REFERENCES


ANNEX A:

ADOPTED MODEL GEOMETRY PLANSET